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AN EXPERIMENTAL AND THEORETICAL INVESTIGATION OF THE CLOGGING OF A RAPID SAND FILTER

by

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To

Professor Thomas R. Camp

in appreciation of all he has done for me

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AN EXPERIMENTAL AND THEORETICAL INVESTIGATION OF THE CLOGGING OF A RAPID SAND FILTER

1. INTRODUCTION

Rapid sand filters are now commonly used in water purification practice to remove suspended solid material from the water. The design of these filters has always been arbitrary because no theories or laws have as yet been established from which one may predict the removal of floc of different characteristics by different sizes and gradings of sands, depths of filter, and rates of filtration.

The object of this research has been the investigation of the clogging of a rapid sand filter. Particular attention has been devoted to the time rate of removal of solid matter from the water in different layers of the filter and the variation of this removal throughout the length of the run. The time rate of increase of lost head in these layers, caused by the removal of the solids, has also been studied.

A filter used in water treatment consists of a bed of sand through which the water usually flows vertically downward. The rate of filtration is generally kept constant. In the water being filtered are particles of extraneous solid matter which it is desirable to remove before the water may be used for practical purposes. This removal of solid matter is, therefore, the primary purpose of the filter.

Figure 1 illustrates how the filter operates. The bed contains sand grains and also spaces between the grains. When the filter is clean, these spaces are occupied by water. As the water containing

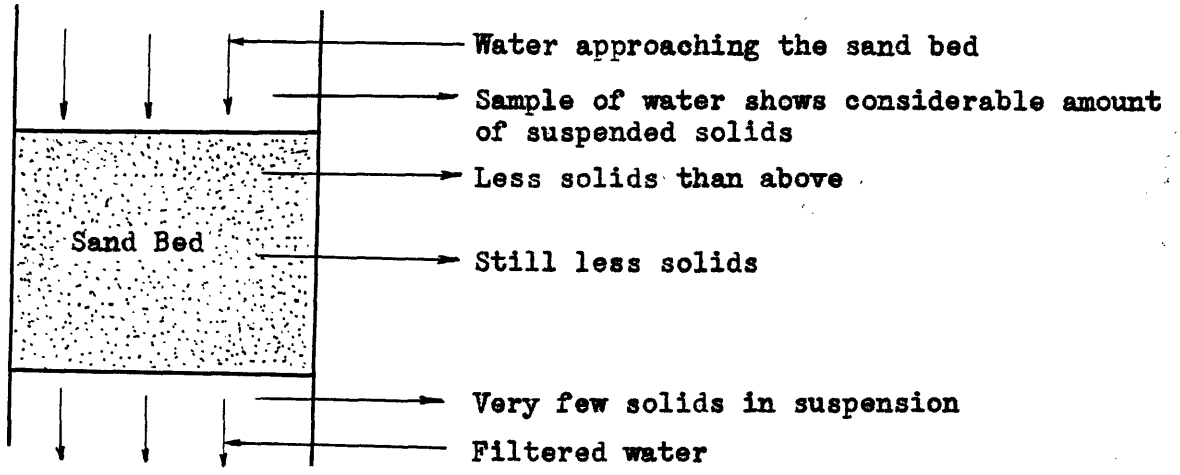


Fig. 1

solids passes through the filter, some of these solids remain in the pore spaces between the sand grains. This process of the filling up of the pore spaces by removal of solids from the water is spoken of as the clogging of the filter. The diagram shows how the solids content of the water decreases as the water proceeds through the filter. At the effluent, few solids are left in suspension. Not only suspended matter is removed, but also bacteria and colloidal particles such as organic coloring matter.

Several theories have been advanced by different authors [1,2], as to the manner in which this matter is removed in the filter. Among these theories are the following:

a) Mechanical straining. This removes the particles which are too large to pass through the spaces between the sand grains.

b) Sedimentation. If the velocities are not too great to prevent settling, the particles of suspended matter will deposit on the upper surfaces of the sand grains.

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1. Camp, T. R., Notes on Sanitary Engineering, p. 121

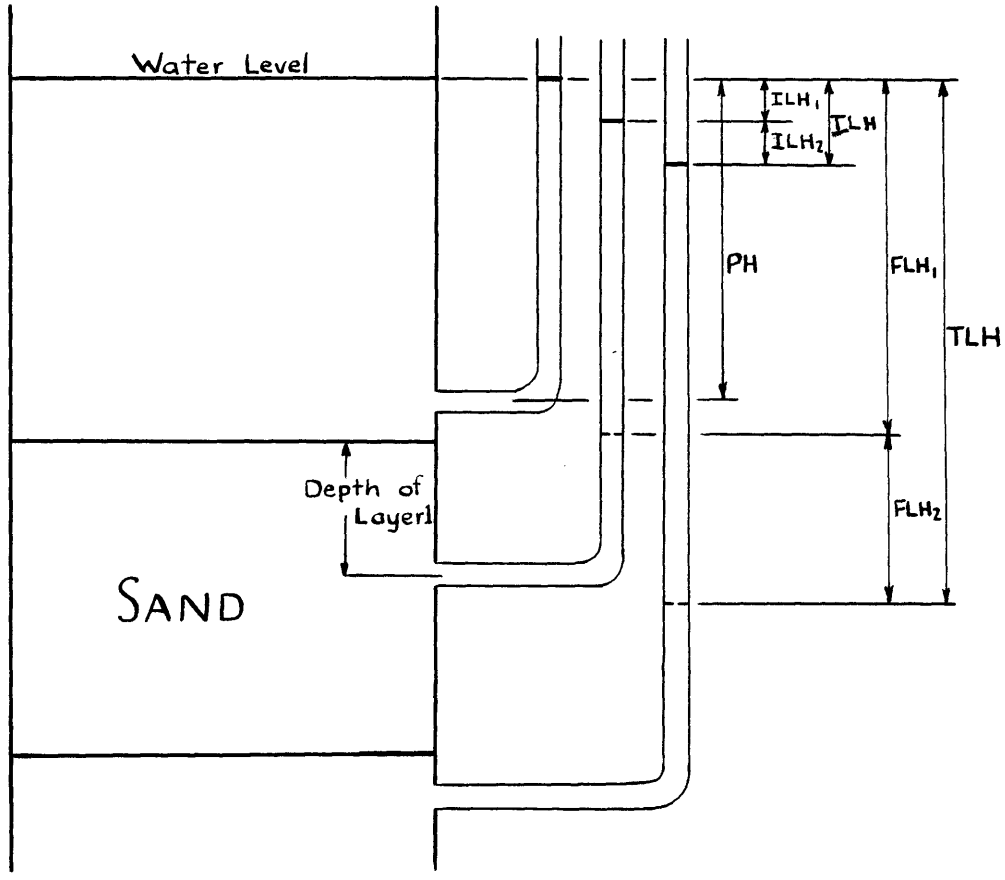
2. Babbitt, H. E., and Doland, J. J., Water Supply Engineering, p. 574

c) Adsorption. Some persons believe that the surfaces of the sand grains are negatively charged. A certain amount of the matter suspended or dissolved in the water is also charged. The particles which are positively charged may be attracted to the negatively charged sand grains and adsorbed on the surface of the sand, neutralizing the charges of both particles and sand grains. A saturation point is reached when a sand grain is entirely neutralized. The result is a neutral, thinly coated grain of sand.

d) Adhesion. Colloidal particles experience a rapid motion known as Brownian movement, in which they appear to be bombarded in all directions. On account of the narrow interstices between the sand grains, Brownian movement may bring these colloids in contact with the neutral surfaces of the coated sand grains. They will be held there until a greater force displaces them.

e) Biological Agencies. Bacteria and other micro-organisms will be removed by the sand grains along with the other solid matter. When organic matter is present for food, it will be eaten by these organisms and thus removed. Generally, there are not enough organisms present to contribute any appreciable removal in filters used for water treatment.

The term "lost head" is frequently used in this thesis. Piezometers are inserted at various depths in the filter as shown in Fig. 2. The difference between the level of the water in any two piezometers is equal to the lost head in that section of the filter located between the two depths at which the piezometers are inserted.



PH Pressure Head

ILH Initial Lost Head - beginning of run

FLH Final Lost Head - end of run

TLH Total Lost Head in Filter

Subscript indicates number of layer

Fig. 2

The resistance to the flow of water past the sand grains causes the drop in pressure between various depths in the filter. When the filter is clean, the velocity of the water through the pores is relatively low, and the corresponding lost head is low. As the filter clogs, there is less volume of pore space to pass through, and the velocity of flow must increase if the same quantity of water is to be filtered per unit of time. In this discussion, the time rate of flow through the filter will be considered constant, since this is usually the case in actual practice. This increase of velocity causes an increase in the lost head between different depths. Figure 2 shows how the lost head increases when clogging takes place.

In general, there are two types of filters in operation at water treatment plants; the slow sand and the rapid sand filter, The first to be developed was the slow sand filter. Without previous treatment, the water was applied to these filters and filtered at rates varying from 2 to 8 million gallons per acre per day. For a large city, this rate of filtration required extensive areas of sand beds which are expensive, both in cost of land and cost of filters. Engineers investigated the use of higher rates of filtration in order to reduce the area required. Eventually they found that by a proper treatment of the water prior to filtration efficiencies of removal of turbidity, color and bacteria could be obtained with a rate many times greater than that previously used. Thus, the rapid sand filter was developed. The essential differences between slow and rapid sand filters are the following:

1. Rate of filtration
2. Character of solid material in the water applied to the filter

3. Sizes of sand grains
4. Method of cleaning the filter
5. Stratification of the sand

The rate of filtration in the slow sand filter ranges from 2 to 8 million gallons per acre per day (M. G. A. D.) while the rapid rate ranges from 100 to 180 M. G. A. D. [1]

With the slow sand filters the water is not ordinarily treated with any chemicals before filtration. But with the rapid sand filters the water is always treated with chemicals which will coagulate to form jelly-like floc particles. Solids in the water, when coming in contact with these floc particles, adhere to them. By passing slowly through a settling basin, most of these floc particles are removed from the water by the filter.

Sand grains are usually characterized by two terms: effective size and uniformity coefficient. A sieve analysis is made of the sand to be used and a curve is drawn with percent passing the different sizes of sieve openings plotted against the size of sand grain as determined by the sieve opening. From this curve two sizes are noted: that size of the grains, below which 10 percent of the sand, by weight, is smaller (the 10 percent size), and that size of the grains, below which 60 percent of the sand, by weight, is smaller (the 60 percent size). The 10 percent size of the sand in the filter is known as the effective size. The uniformity coefficient is obtained by dividing the 60 percent size by the 10 percent size.

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1. Babbitt, H. E. and Doland, J. J., Water Supply Engineering, p. 580

For slow sand filters the sands range in effective size from 0.20 to 0.45 mm., with uniformity coefficients from 1.5 to 4.0. These are relatively fine sands with a considerable range in the grading. For rapid sand filters the sands are usually coarser, with a more uniform grading. Thus, the effective sizes range from 0.35 to 0.50 mm., with a uniformity coefficient from about 1.4 to 1.7.[1]

Slow sand filters are usually cleaned by scraping and removing the surface layer to a depth of from $\frac{1}{2}$ to $1\frac{1}{2}$ inches, and washing the sand in a special washing machine. This is done at intervals from one to six months, depending on the material in the water and the rate of filtration. With the rapid sand filter the method of cleaning is radically different. Water is forced upward through the bed at a rate high enough to hold the sand in suspension. This separates the sand grains. The flocculent material is washed off the grains and is carried out by the high velocity of the wash water.

In the construction of a slow sand filter, the sand is dumped into the bed with no attempt made to stratify the sand. When the filter is in operation, grains of all different sizes may be found throughout the depth of the bed. In the rapid sand filter, the washing process stratifies the sand, with the smaller grains at the top.

The force which suspends the sand grains is the frictional resistance to the upward passage of the water through the sand bed. When the velocity of the water past a sand grain becomes sufficient to cause a frictional resistance equal to the weight of the sand grain in

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1. Babbitt, H. E., and Doland, J. J., Water Supply Engineering, p. 560

the water, the grain becomes suspended. The heavier grains require a greater velocity to suspend them and this is brought about by the grains being closer together, giving a smaller area of flow, therefore a greater velocity. The lighter particles will be forced upward by the higher velocities to a section of the bed where the velocity causes a frictional resistance on each particle equal to its weight. The bed will then be stratified, with the lighest grains on top and the heaviest at the bottom. Assuming that all particles have the same specific gravity, which is usually the case with the sand in any one filter, this means that the bed will be stratified with the smallest grains at the top and the largest grains at the bottom of the bed.

Through a slow sand filter the loss of head may increase from a fraction of a foot when the filter is clean to as high as 6 or 7 feet when it is ready to be cleaned again. It is common belief that the greater portion of the frictional resistance to the flow of water through the filter occurs at the surface of the sand in the layer of matter which has deposited there. This layer of material has been called by the Germans the "schmutzdecke," or mud blanket. The effect of the sand grains below the surface is believed to be very small, but increases with increase in the size of the grains. Some persons believe that this coating on top of the filter is not only necessary for successful operation of the slow sand filter, but for the rapid sand filter also. In writing about rapid sand filters, Tyler states that "the effective filtering stratum consists of the floc layer on top of the sand and the fine surface sand which supports it."[1] Still the head

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1. Tyler, R. G., Danielson, W. A., Le Bosquet, M., Jr., Head Losses in the Rapid Sand Filters at Cambridge, Mass. Jour., N. E. Water Works Assn. 40: p. 341, Sept. 1926

loss curves accompanying his report show an increase in the lost head during one run (a run extends from the time the filter is started after washing until it is shut off for washing again) of one foot in the layer extending from a depth of 0.55 feet to 1.25 feet. The increase of lost head in this layer indicated penetration of floc to this depth and, therefore, that this layer too is part of the "effective filtering stratum."

Investigations by Armstrong[1] and others, and experiments by the author, have shown that the deposit on the surface of the filter is not essential to efficient filter operation. A good effluent may be obtained by using sand grains which are coarse enough to allow all the floc to penetrate into the bed itself. The loss of head may increase from around one foot when the filter has just been washed to as high as 8 or 9 feet at the end of a run, the limiting value being determined either by that allowed by construction features in the plant or the character of the effluent.

As stated on page 1, the object of this research has been the investigation of the clogging of a rapid sand filter. Particular attention has been paid to the time rate of removal of solid material from the water in different layers of the filter and the variation of this removal throughout the length of the run. The time rate of increase of lost head in these layers has also been studied.

Among the factors which may influence the depth to which coagulated matter will penetrate, the rate per unit time at which it will be removed

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1. Armstrong, J. W., Filter Sand. Journ. Am. Water Works Assn. 23: p. 1299, Sept. 1931

in various layers, and the time rate of increase of lost head are the following:

1. Size of sand grains
2. Shape of sand grains
3. Variation of sand size with depth
4. Character of flocculated matter
5. Concentration of flocculated matter
6. Amount of flocculated matter already in the bed at each depth
7. Rate of filtration
8. Temperature of water
9. Porosity of filter bed
10. Consolidation of the bed during the run

The degree of importance of these variables is not all the same. Baylis states, "The depth to which coagulated matter will penetrate depends on several conditions, the most important of which are the size of sand and the character of the coagulated matter."[1] As a result of the studies made by the author, he believes that the factors listed above are the ones influencing the clogging of a rapid sand filter.

These studies have considered an important phase of the problem of filtration. As far as the author knows, no article has been published which is the result of studies on the amount of flocculated matter removed per unit of depth throughout the depth of the filter, and the variation of the time rate of this removal during the run. A knowledge of the time rate of clogging, the amount of matter removed per unit of

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1. Baylis, J. R. A Study of Filtering Materials for Rapid Sand Filters. Water Works and Sewerage 81: p. 167, May 1934

depth, and the time rate of increase of lost head accompanying this clogging are essential for the development of a complete theory of filtration.

The ultimate object of investigators in the field of filtration is to develop a complete theory of filtration which will correlate the variables listed on page 10, or as many more or less variables as are found to effect the clogging of a rapid sand filter. When such a theory is developed, the engineer designing a filter plant will be able to predict the performance of the filter when using different depths of sand bed, sizes and grading of sand, rates of filtration, and character of flocculated matter. Knowing the results which will be obtained with different combinations of these variables, he may compute the combination of factors which will give him the most economical plant from the standpoint of initial cost and cost of operation.

Because the rate of filtration generally used is around 125 million gallons per acre per day, and the depth of sand between 24 and 30 inches, with certain sizes and grading of sand, it does not follow that these are the optimum values of the various factors. They have worked well in the past and, therefore, are used in present designs. But with the advent of a workable theory of filtration, a rational design may be made in which these factors may have far different values. William E. Stanley, Chairman of the Committee on Filtering Materials of the Sanitary Engineering Division, American Society of Civil Engineers, has written: "It is not unlikely that the present practice might be modified as more definite information becomes available of the effects of various physical characteristics, such as size and gradation of filter sand."[1]

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1. Stanley, W. E. Filtering Materials for Water Works. Journ. Am. Water Works Assn. 23: p. 1286, Sept. 1931

It is hoped that these studies will throw new light on the subject of filtration and that a better understanding of the performance of rapid sand filters will result. Perhaps this research may be considered as an important step toward the development of a complete theory of filtration.

2. SCOPE OF INVESTIGATION

The complexity of the problem of filtration necessarily limits the field which may be covered by one person in the course of a year's research. For a complete theory of filtration, research would have to be conducted over a considerable period of time in order to observe the effect of all the variables over the entire range met with in actual filtration practice.

With the facilities for research at the filtration plant of the City of Providence, Rhode Island, where the experiments described herein were conducted, it was possible to use the water coming from the plant's sedimentation basins. The floc particles in this water varied both in size and in concentration from time to time. It was decided to focus the attack on these two variables and to observe their effect on the time rate of deposit of solid matter in various layers of the filter, and on the time rate of change of lost head in these layers. It was not possible to control the temperature of the water or the consolidation of the bed during the run. But the factors which could be controlled, namely, size, shape and grading of sand grains, rate of filtration, and initial porosity of the bed were made constant, during the course of these experiments.

The temperature of the water did not vary much more than a degree from four degrees Centigrade. All values of lost head were corrected for any difference in temperature.

The depth of bed was made the same after each wash. A very small portion of the matter deposited in the bed could not be washed out, but

the difference it made in the porosity of the bed was negligible. This is shown by the fact that the lost head immediately after washing was the same after runs aggregating 1200 hours of operation as it was when the sand was first put in the filter. Thus the average porosity of the bed was the same at the beginning of each run.

No attempt has been made in these studies to determine the relative importance of the different theories of actual removal of solid matter by the sand grains, namely, mechanical straining, sedimentation, adsorption, adhesion, and biological agencies. Study has been limited to the collective result of these actions, namely, the clogging of the rapid sand filter.

3. THEORETICAL CONSIDERATIONS

In a recent paper entitled, "Fundamental Factors Governing the Stream-line Flow of Water through Sand,"[1] Fair and Hatch developed the following equation for the hydraulic gradient of the flow of clean water through clean sand:

$$\frac{h}{l} = \frac{K\mu}{w} v \frac{(1-f)^2}{f^3} \frac{S^2}{d^2} \qquad (1)$$

Here h = loss of head through the sand

l = depth of sand

d = geometrical mean diameter of sand grains

k = a filtration constant

u = coefficient of viscosity

w = weight per unit volume of water

v = velocity of approach of water to filter

f = porosity of sand bed

S = Shape factor of sand

This equation holds for a layer of the sand bed in which the grains are of the same diameter. To include the whole of a stratified bed of sand, it will be necessary to integrate the equation over the entire depth of the bed, taking account of the variation of the diameter of the sand grains with depth.

This equation appears to be rational. The authors have published this and several other related equations and have given values, or means for determining the values, of the constants. But as yet they have not published any data to show how their equations check with experimental

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1. Fair, G. M., and Hatch, L. P. Journ. Am. Water Works Assn. 25: p. 1559, Nov. 1933

results. In their paper they have stated the following: "Series of tests made by the authors, but not recorded in this paper, show close agreement between theory and experiment both in regard to individual variables and collective effects." Before the equations may be wholly accepted, it must be shown that they hold over a wide range of shapes and sizes of sand and rates of filtration.

The constant, K, takes account of the relative surface areas of the sand grains in contact with flowing water. In the washing process, where the grains are suspended, the value of K is given as 4. In filtration of clean water through clean sand, where portions of the surface areas of the grains are in contact with each other, and not with the water, the value of K is given as 5. These factors must also be checked with experimental results.

The coefficient of viscosity, μ, is a function of the temperature. Knowing the temperature of the water, μ may be obtained from the curve shown in Fig. 3. The points for this curve were plotted from experimental values determined by Bingham and Jackson.[1]

Equation 1 shows that the lost head in the filter is directly proportional to the coefficient of viscosity. In the results of the experiments described in this thesis, this fact was used to convert the values of the lost head at different temperatures to one temperature, namely, four degrees Centigrade. This was done by adjusting the lost heads in proportion to their respective coefficients of viscosity.

The diameter of the sand grains, d, is the geometric mean diameter as determined by taking the square root of the product of the sieve

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1. Handbook of Chemistry and Physics, Chemical Rubber Publishing Co. 16th Edition p. 857 1931.

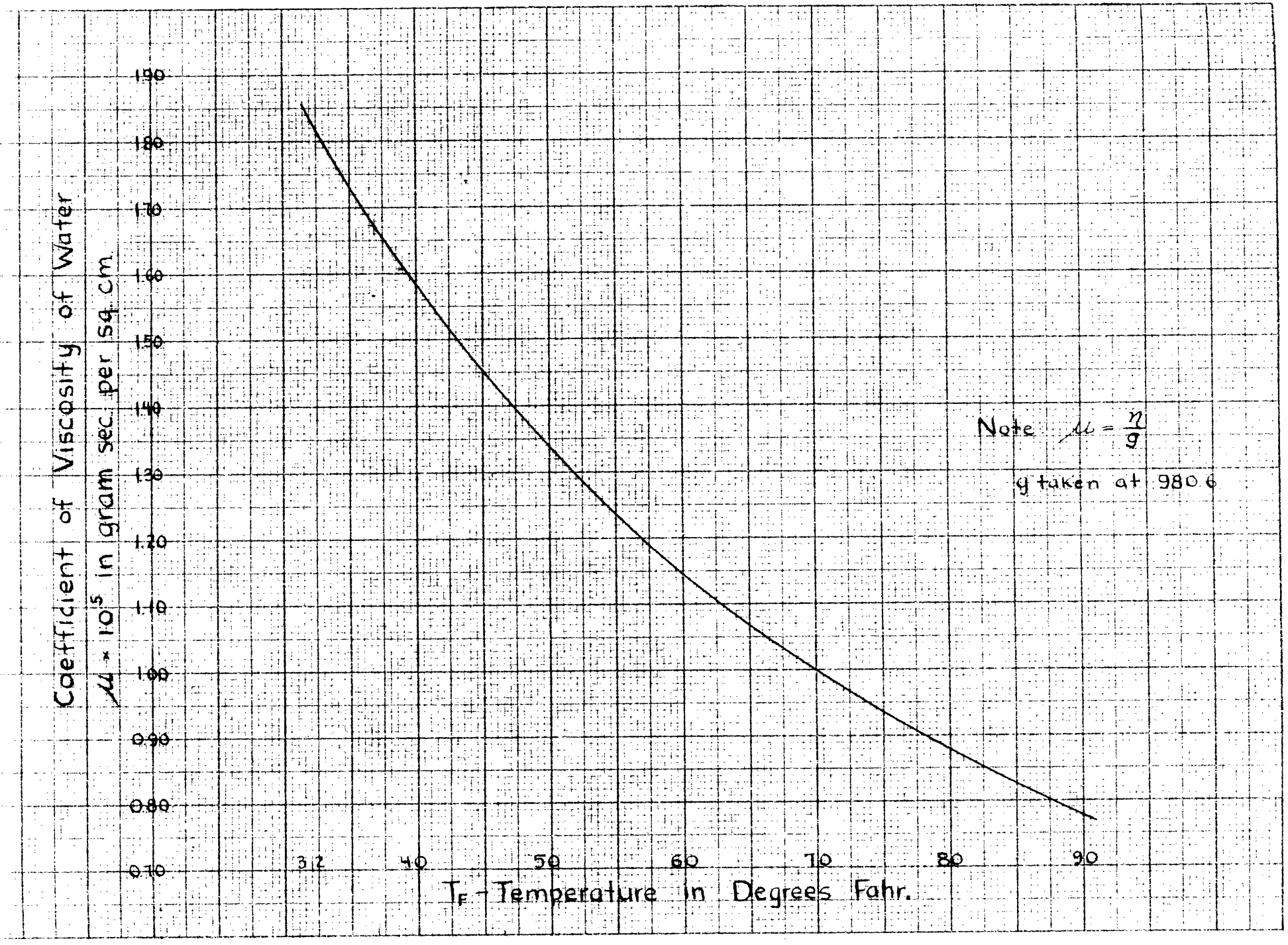

Coefficient of Viscosity of Water
$\mu \times 10^5$ in gram sec. per sq. cm.
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
32
40
50
60
70
80
90
T_F - Temperature in Degrees Fahr.
Note $\mu = \frac{\eta}{g}$
g taken at 980.6

FIG. 3

openings of two adjacent sieves, one through which the sand grains have passed, and the other upon which they have been retained.

The shape factor of the sand, S, is a factor, which, when multiplied by the reciprocal of the diameter of a sand grain, will give the area-volume ratio of the sand grain. The authors have given two methods for determining this. One is to examine the grains under the microscope and to compare their shapes with those illustrated in the paper, for which values are given, and choosing a value which seems to correspond with the shape of the grains under the microscope.

A method is also given for determining the value of the shape factor experimentally. The sand is backwashed and readings are taken of the pressure at the top and bottom of various layers of the expanded bed, in order to obtain the lost head, h, in these layers. The depth, l_e, of each layer must be accurately measured. The area of the filter and also the quantity of wash water flowing per unit time must be known in order to obtain the velocity of wash water. The temperature of the water is measured to obtain the coefficient of viscosity. These values are then substituted in the following equations:

$$\frac{h}{l_e} = \frac{R-r}{r}(1-f_e) \qquad (2)$$

$$\frac{f_e^3}{(1-f_e)^2} = \frac{K\mu}{R-r} v \frac{S^2}{d^2} \qquad (3)$$

Here h = lost head in distance l_e

l_e = height of expansion

R = specific gravity of sand

r = specific gravity of water

f_e = porosity during expansion

K = a constant equal to 4 in washing

u = coefficient of viscosity

v = velocity of wash

d = geometrical mean diameter of sand

S = shape factor of sand

This latter method of determining the shape factor of the sand grains has been used for the experiments described in this thesis.

Although developed for the filtration of clean water through a clean sand bed, the author of this thesis believes that the application of Equation 1 may be extended to cover the flow of water containing flocculated matter through a filter undergoing clogging. He further believes that it may be considered the fundamental equation of filtration, giving the hydraulic gradient at any point in the filter at any instant during the process of clogging the filter. The factors will naturally be different from those in the case of clean sand and clean water. At any point in the filter, a given set of values for K, f, S, and d, may hold for an instant, but will change as the filter continues to clog. Thus, the equation may be used to give the instantaneous hydraulic gradient at any point in the filter at any time during the process of clogging, provided the proper factors be substituted.

During the process of clogging, the following factors in

Equation 1 may vary: K, f, S, and d. As the pores between the sand grains are filling with deposited material, a smaller portion of the area of each sand grain comes in contact with the flowing water. This may increase the value of K.

By definition, the clogging of a filter signifies a decrease in the porosity of the bed. Therefore, one of the important variables in this equation is the porosity.

The diameter of the sand grain itself is, of course, fixed. But the floc particles may coat these grains, either uniformly or unevenly, and thus have the effect of increasing the diameter of the grain.

The shape factor may possibly remain the same if the sand grains are coated uniformly, but if coated unevenly, the shape factor may experience a considerable change.

A factor which may influence the increase of lost head during clogging is the consolidation of the bed during the run. In the experiments described herein, the consolidation occurred gradually during each run and at the end of the run averaged about 0.8 percent of the total depth of the bed. Although the consolidation is not very great, it may have its effect on increasing the lost head in the filter. The equation does not take into account this consolidation.

One of the objects of this thesis has been the investigation of the effect on the rate of increase of lost head with time, of the consolidation of the bed and the factors represented in Equation 1, namely, K, f, S and d. This equation of filtration will be used as a basis for the interpretation of the experimental results obtained in

these studies in an attempt to evaluate the relative effects of these factors.

A complete theory of filtration must take account of these factors and their variation with time during the process of clogging the rapid sand filter.

4. PROPERTIES OF SAND

a. Sieve Analysis

The sand used throughout these tests was white silica sand from Ottawa, Illinois. Using Tyler standard sieves and a twenty-minute sieving period in a Ro-Tap mechanical shaker, the following analysis was obtained:

Mesh	Sieve Opening-mm.	Grams Retained	Percent	Percent Passing
20	0.833	0.0	0.0	100.0
24	0.701	41.0	4.15	95.85
28	0.589	165.25	16.7	79.15
32	0.459	550.0	55.6	23.55
35	0.417	214.9	21.7	1.85
42	0.351	18.35	1.85	0.0
	Total	989.5	100.0	

This sieve analysis curve, plotted to natural scales, is shown in Fig. 4. It is customary to plot the analysis with the percent passing to a natural scale and the screen openings to a logarithmic scale, but with sand such as this, confined within such a small range of diameters, this type of plot loses its value. Armstrong, in reporting his analyses on filter sands, uses natural scales throughout.[1]

The short, sharply curved lower portion of this analysis curve is caused by the fact that most of the finer material has been sieved out. Some of the coarse material has also been sieved out, but not to such a great extent as the fine.

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1. Armstrong, J. W. Journ. Am. Water Works Assn. 23: p. 1296, Sept. 1931

Some operators and investigators choose to calibrate their sieves according to a method developed by Hazen.[1] The size of the grain is defined as the diameter of the sphere of equal volume. The last and largest particles passing are taken to represent the size of separation of each sieve. A convenient number of these particles are collected, weighed, and counted, to obtain the average weight of the particles. Hence, if the volume is $\frac{W}{S}$, the diameter is:

$$d = \sqrt[3]{\frac{6}{\pi}\,\frac{W}{S}}$$

A calibration of the sieves by the above method is shown in Table 6 in the Appendix.

The following analysis is obtained:

Mesh	Size of Separation mm.	Percent Passing	
20	0.863	100.00	- 3.96
24	0.778	95.85	- 1.96
28	0.666	79.15	15.52
32	0.545	23.55	4.615
35	0.495	1.85	.936
42	0.422	0.00	0

This sieve analysis, plotted to natural scales, is shown in Fig. 4.

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1. Hazen, Allen. Report of Mass. State Board of Health. p. 550, 1892

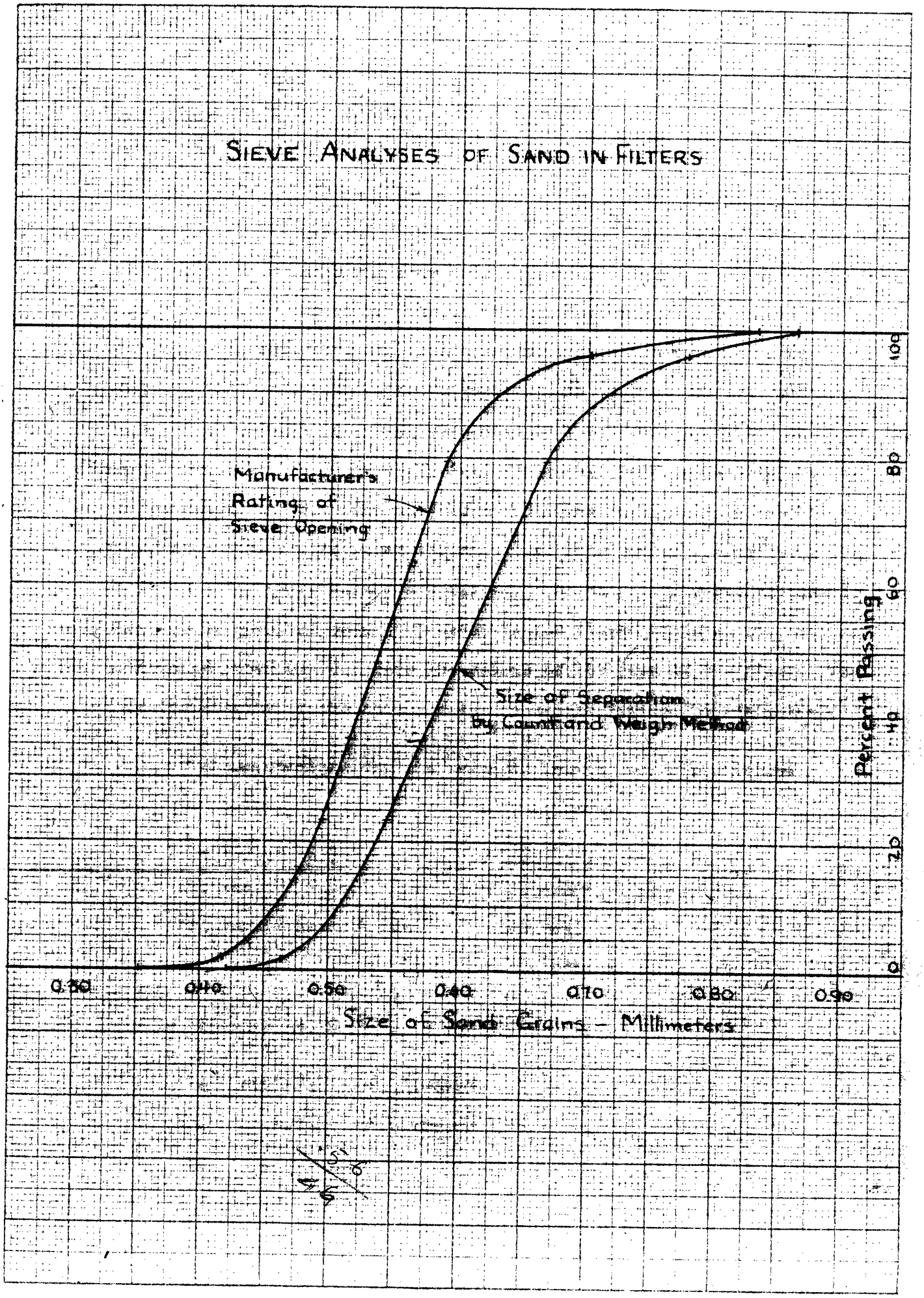

SIEVE ANALYSES OF SAND IN FILTERS
Manufacturer's Rating of Sieve Opening
Size of Separation by Count and Weigh Method
100
80
60
40
20
0
Percent Passing
0.30
0.40
0.50
0.60
0.70
0.80
0.90
Size of Sand Grains - Millimeters

FIG. 4

b. Specific Gravity

The specific gravity of the sand was determined by using a 50 c.c. weighing bottle. The bottle was first weighed empty. Then about 20 c.c. of sand were poured in and the sand and bottle weighed to obtain the dry weight of the sand. Distilled water was then poured in and the water was allowed to boil by placing the bottle in a bath of boiling water. This expelled the air trapped between the sand grains. The bottle and contents were allowed to cool to room temperature, after which more water was poured in, this time up to the 50 c.c. mark. Then the sand, water and bottle were weighed. This weight minus the weight of the dry sand and the bottle gave the weight of the water. The temperature of the water was read, and from tables, the density of water at this temperature was obtained. The weight of water divided by the density gave the volume of water in the flask. This volume, when subtracted from 50 c.c., gave the volume of the sand in the flask. The specific gravity of the sand equals its weight divided by its volume.

The computations for the specific gravity are given below:

Wt. bottle	= 24.2950 gm.
Wt. bottle + sand	= 78.3610
Wt. sand	= 54.0660
Wt. sand + bott. + wat.	= 107.8682
Wt. water	= 29.5072 gm.

Density of water at 22.1° c. = 0.99778 gm./c.c.

log	29.5072	= 1.46993
log	0.99778	= -9.99903 - 10
log	vol. water	= 1.47090
	c.c. water	= 29.573 c.c.

50.000 - 29.573 = 20.427 c.c. sand

log 54.066 = 1.73293

log 20.427 = 1.31021

log sp.gr. = 0.42272

Specific gravity = 2.647

The value of 2.65 will be used as the specific gravity of this sand.

c. Porosity

For all runs, the average porosity was the same after each wash. This is shown by the fact that the initial total lost head in the bed was the same after runs aggregating 1200 hours as it was when the sand had just been placed in the filter. After each wash, the depth of the filter was made equal to 23.5 inches, or 1.96 feet. The area of the filter was 2.02 feet. Therefore, the total volume of the filter amounted to 3.96 cubic feet. 387.5 pounds of sand occupied this volume. The specific gravity of the sand being 2.65, its density is equal to 2.65 x 62.43, or 165 pounds per cubic foot. The actual volume occupied by the sand amounted to $\frac{387}{165}$, or 2.34 cubic feet. The volume of voids was equal to 3.96-2.34, or 1.62 cubic feet. The porosity was, therefore, equal to $\frac{1.62}{3.96}$, or 40.8 percent.

The average porosity of the bed after each wash was 40.8 percent.

d. Shape Factor of Sand

The shape factor of the sand was determined experimentally by the method developed by Fair and Hatch outlined on page 18. The readings taken during the process of backwashing are given in Table 5 in the Appendix. Immediately following this table are the calculations for the shape factor.

With the graded sand in the bed, and the location of the piezometer tubes in the filter, it was possible to get only an average shape factor. Fair and Hatch state that the shape factor is the same for the whole depth when using one kind of sand,[1] but this must be checked by experiment before it can be accepted as a valid assumption.

The average shape factor of the sand used in the experiments described herein is 6.20.

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1. Fair, G. M., and Hatch, L. P., Journ. Am. Water Works Assn. 25: p.1556, Nov. 1933

5. CHARACTERISTICS OF FLOC

The size of floc particles and their concentration in the water are very important in determining the length of filter runs. Baylis emphasizes this in the following statement: "The first step in the development of a theoretical equation for computing a predicted filter performance must be a means of computing the clogging tendency of the suspended material in the water."[1]

The characteristics of floc are considerably different at different plants, and vary over quite a range at one plant. The dissolved, colloidal and suspended matter in the water, the chemicals added, the hydrogen-ion concentration (pH) of the water, coagulation facilities, temperature, and length of settling period all have their influence on floc characteristics.

At the filtration plant of the City of Providence, the raw water total solids content is about 50 ppm. Ferric sulphate is added as the coagulant and lime is added to bring the pH in the mixer to a little over 10. Good mixing and the high pH give good flocculating conditions. The length of settling period is exceptionally long, from 48 to 96 hours. Most of the floc settles out in the sedimentation basins, and as a result, the floc particles going to the filter are very small, generally invisible to the naked eye.

The size of floc particles can be determined by microscopic examination. Generally, the floc particles were so small and so nearly transparent that they could not be seen under the 100-power microscope without straining. A few drops of carbol fuchsin stain

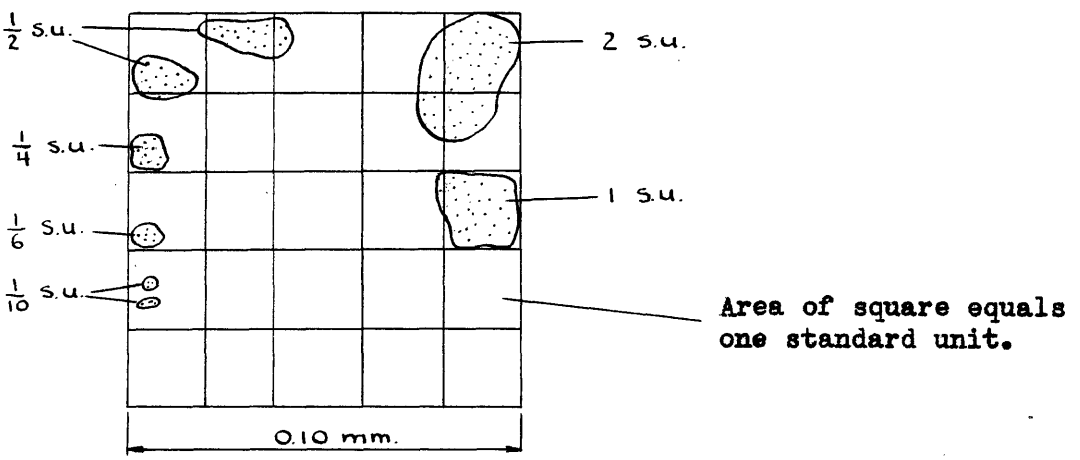
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1. Baylis, J. R., Water Works and Sewerage 81: p. 354, October 1934

added to the water made these particles visible by coloring them a deep red.

The ocular micrometer of the microscope gives a field 1 mm. square, with rulings 0.1 mm. apart running perpendicular to each other and forming squares 0.1 mm. on a side. One of these squares in the center of the field is further subdivided into squares 0.02 mm., or 20 microns, on a side. The area of one of these squares is 0.0004 sq. mm., or 400 sq. microns, and is known as one standard unit. In measuring the area of the floc particles, the standard unit was used as the unit of area.

The sketch in Fig. 4A shows this 0.1 mm. square subdivided into 25 standard unit squares. Floc particles are shown in these squares to give an idea of their shape and size as seen under the microscope. The dots represent black granules and the shaded portion represents the jelly-like substance of which the floc is composed. The floc particles are here magnified 500 times.



Floc Particles

Fig. 4A

A method similar to that used in the examination of water for microscopic organisms was applied in determining the areas of the floc particles. A portion of a sample of water was taken in a pipette and placed into a counting cell. The cell was placed under the microscope, and a number of fields of view 1 mm. square were examined. The area occupied by each particle in the field was observed. Instead of tabulating the actual observed area of each particle, for ease of computation, certain fractions of standard units were chosen, namely, 1/2, 1/4, 1/6, 1/10, and the particles were tabulated under that fraction nearest the value of its observed area. In each field, the number of particles of each area were counted. This gave a tabulation of the sizes of floc particles and their relative numbers.

It is not known what measure of the size of the floc particles it is possible to link up with a rational theory of filtration. In this thesis, the arithmetical average of the areas will be used as an index of the size of the floc particles. This is arbitrarily chosen for its simplicity. Subsequent research will have to be carried out to find the proper index of the size of the floc particles.

The following is an example of the determination of the average particle size for one sample of water:

Influent to Filter Run 8 10 fields of 1 sq.mm.

Number of Particles	Size st. un.	Total Areas	
2	1	2 x 1.00	2.00
14	1/2	14 x 0.50	7.00
21	1/4	21 x 0.25	5.25
21	1/6	21 x 0.167	3.50
26	1/10	21 x 0.10	2.10
84		84	19.85
			0.236

Average area = 0.236 st. un.

It must be realized that the floc particles visible under the 100-power microscope are not the only ones visible in the water. They actually range from the largest ones down to colloidal sizes. These latter also take part in clogging the filter, but not to such a great extent as the larger ones.

The determination of the concentration of flocculated matter in the water presents a great problem. A floc particle consists of ferric oxide, the solid matter removed by the particle from the water, and considerable adsorbed water. For a given amount of iron, the water content may vary considerably in different particles, thus giving different volumes of floc. Also, for a given size of floc particle, the solids content of each particle, other than iron, may vary over quite a range. The author could not devise a means of determining accurately the concentration of flocculated matter in the water, that is, the total volume occupied by all the floc particles per unit volume of water. As an alternative, an index of the concentration of the flocculated matter in the water had to be sought.

The following alternatives were considered: suspended solids content of the water, turbidity of the water, and the iron content of the water.

The suspended solids test[1] gives the dry weight of suspended solids per unit weight of water. For water in which the concentration of suspended solids is relatively high, this might give good results. But the suspended solids content of the settled water in Providence was only 7.3 parts per million (ppm), and after having passed through the

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1. Standard Methods for the Examination of Water and Sewage p. 27, 7th Ed., 1933

filter, it was almost zero. The result of the test depends on the difference between two small quantities, in both of which there are chances for error. Accuracy could not be expected with this test, so its use was not deemed advisable.

Some experimenters use the turbidity of the water as an index of the concentration of flocculated matter. In the experiments described herein, turbidity readings were taken of the samples for awhile, but no consistent results were obtained. One of the reasons for this is that the turbidity of the settled water was usually less than 1 part per million. With the Hellige turbidimeter used in the tests, it was not possible to get readings closer than 0.05 ppm. For this work, greater accuracy was necessary. Since consistent results could not be obtained, the author ceased taking turbidity readings.

The test for the iron content of the water gives the grams of iron per million grams of water (ppm).[1] It is a comparatively simple test to perform and gives accuracy within 0.01 or 0.02 ppm for the concentrations of iron met with in these experiments, namely, between 0.04 and 0.60 ppm. As stated before, for a given amount of iron, the water content may vary considerably in different particles, thus giving different volumes of floc. But the iron content of a given volume of water may conceivably be a good index of the average concentration of these floc particles. That is, although the ratio of iron to volume may vary with different particles, the average for many particles may be relatively constant. In the settled water at Providence, the number of particles visible under the 100-power microscope ranged between 10,000 and 20,000 per c.c. It is possible that the average iron content

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1. Standard Methods for the Examination of Water and Sewage p. 46, 7th Ed., 1933

per unit volume of these particles might have a fairly constant value and thus permit the use of iron as an index of the concentration of flocculated matter in the water.

Some tests were made to determine the ratio of iron to suspended solids in the water. In the water applied to the filter, the iron content was found to be 0.62 ppm and the suspended solids content was 7.3 ppm. Thus, the iron content was 8.5 percent of the suspended solids content. The average of two other tests, with iron content equal to 0.31 ppm and 0.21 ppm, respectively, was 8.6 percent. These tests show a consistent relation between iron content and suspended solids.

Furthermore, subsequent results described in the latter part of this thesis, using iron as an index of the concentration of flocculated matter, show no inconsistencies or contradictory results. It would appear that this is further indication that the iron content of the water may be used as an index of the concentration of flocculated matter in the water.

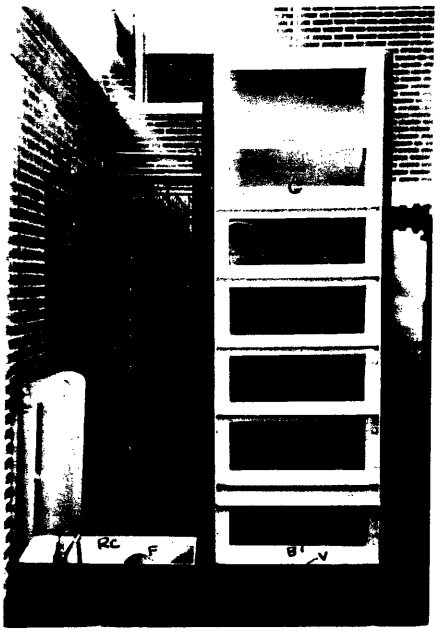
One characteristic of floc particles which would be very difficult to measure, is the strength of the floc. This may well vary with different types of floc. This characteristic may also have its effect on the depth of penetration of flocculated matter. The stronger floc may resist breaking up in the pores of the filter bed, while the weaker floc may easily be broken and the fragments carried deeper into the bed. However, with the floc used in the experiments for this thesis, the particles were not greatly different in size and all were formed with the same coagulant, ferric oxide. It is reasonable to expect that these particles would not vary greatly in strength; at least, not enough to have any appreciable effect on the depth of penetration of the floc.

In the experiments described herein, the average area of the floc particles was used as an index of the size of the particles. The iron content of the water was used as an index of the concentration of flocculated matter in the water.

6. FILTRATION APPARATUS

Two filters were used for these experiments. They were located in the pump room of the filtration plant of the City of Providence. The larger one (Figs. 5 and 6) has an area of 2.02 square feet, rectangular in cross-section, with three sides of metal and one of glass. The sand is supported by a porous plate, which also serves to distribute the wash water.

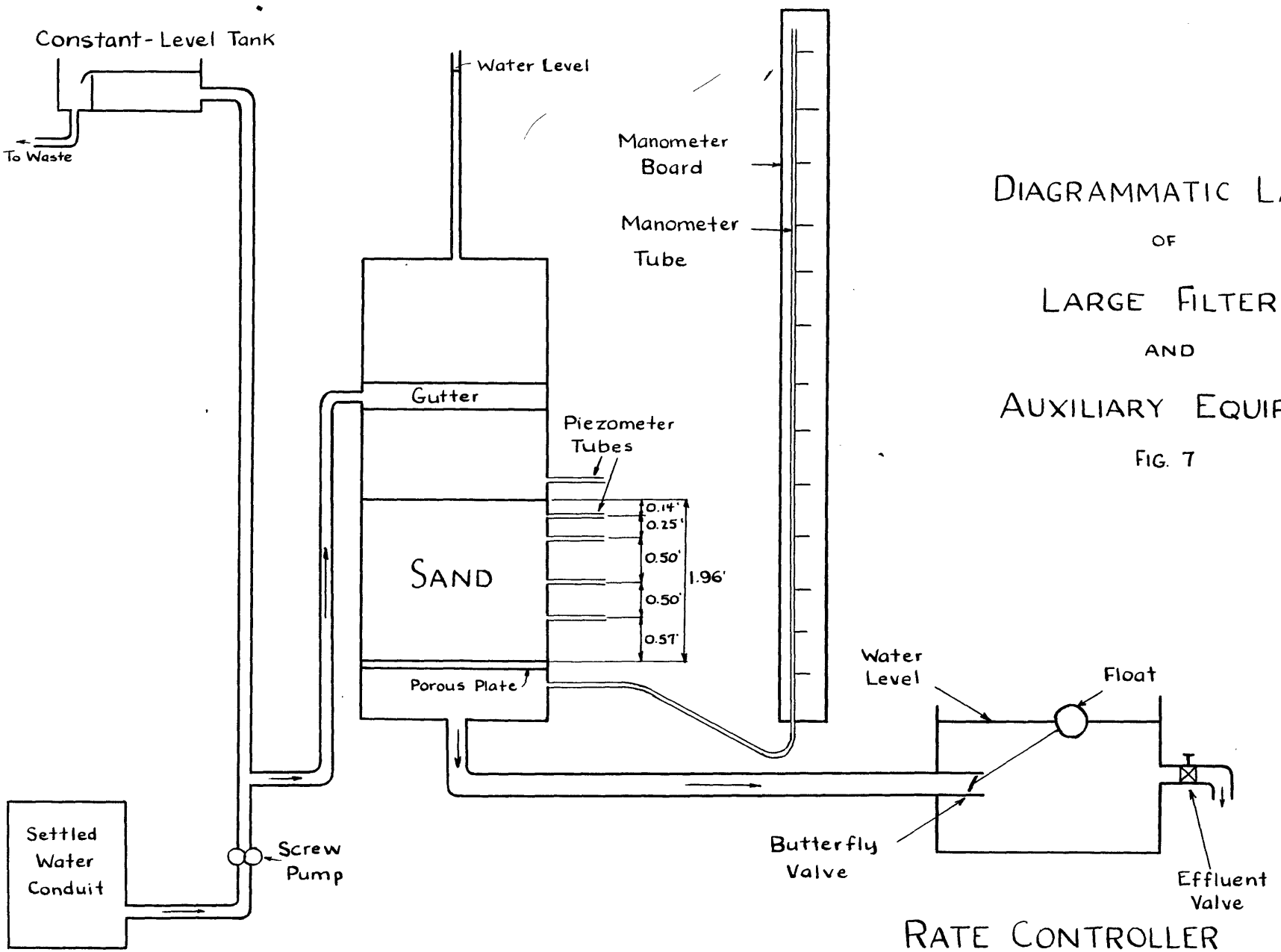
Water was taken out of the settled water conduit and pumped by a small screw pump to the influent of the filter. A constant pressure on the filter was secured by means of a constant-level tank connected as shown in Fig. 7. An overflow weir which continually wasted a small amount of water kept the level in the tank constant. This tank is marked T in Fig. 10. From the influent pipe (I.P., Fig. 6), the water was led into the filter tank through the wash water gutter (G, Fig. 5). After passing through the sand, the filtered water went through the effluent pipe (E.P., Fig. 6) to the rate controller tank (R.C., Figs. 5 and 6). The rate controller (Fig. 7) consists of a tank into which projects the effluent pipe. In the end of this pipe is a butterfly valve which controls the effluent rate. The opening of this valve is controlled by the float (F) which floats on the surface of the water in the tank. The rate of flow out of the tank is controlled by the opening of the valve (E.V.), which acts as an adjustable orifice, and the level of water in the tank, the head on the orifice.



Large Filter
Fig. 5



Large Filter
Fig. 6



DIAGRAMMATIC LAYOUT OF LARGE FILTER AND AUXILIARY EQUIPMENT

FIG. 7

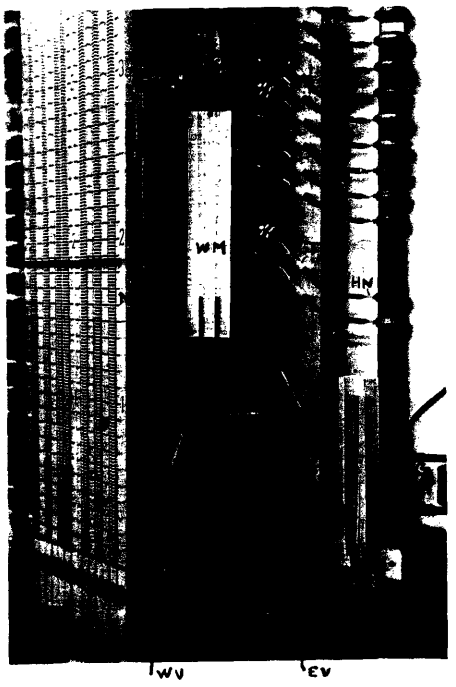


Fig. 8

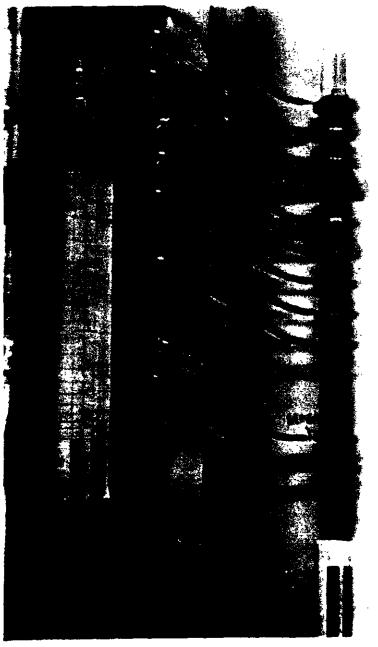
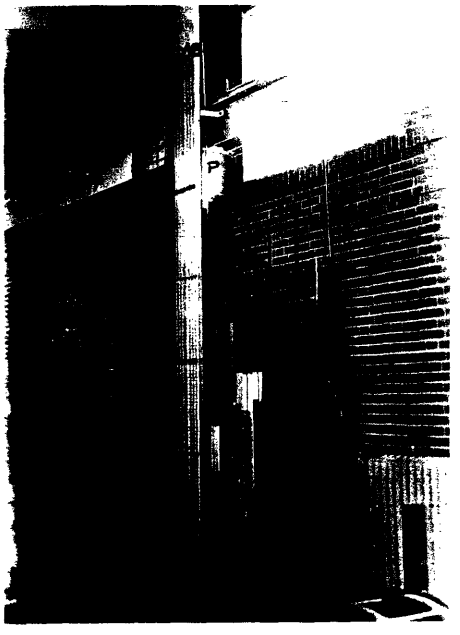


Fig. 9

Small Filter and Auxiliary Equipment



General View of Small Filter,
Manometer Board and
Constant-Level Tank

Fig. 10

Wash water was supplied from the plant's wash water tank. It entered at the bottom of the filter through the valve (V, Fig. 5), was dispersed laterally by the baffle (B), and further distributed by the porous plate, passed up through the sand bed and was collected in the gutter (G), whence it went to waste through the waste pipe (P, Fig. 5).

Piezometer tubes are provided in the filter and are spaced as shown in Figure 7. They may also be seen projecting at the left of the filter in Figures 5 and 6. Through these tubes, the samples of water were taken. By connecting each of these piezometer tubes to one of the manometer tubes on the manometer board (M.B., Fig. 10), the pressure at each of these points in the filter could be determined.

The temperature of the influent water was read on a thermometer located in the influent pipe, just before entering the filter (T.I., Fig. 6). The effluent thermometer was located in the chamber between the porous plate and the bottom of the tank (T.E., Fig. 5.).

The smaller filter was arranged in a battery of filters as shown in Figure 8. The other filters did not take part in these experiments. Figures 8 and 9 show the filter in detail. It is circular, with an area of 9.09 sq. cm., and consists of a number of units of various lengths. These units are composed of a piece of glass tubing with a steel header at each end. The headers are cemented to the glass with copper phosphate. Each header is so constructed that a screen may be held in place at each end to keep the sand separate in each unit. A hole through each lower header provided a piezometer connection which is made by a short hypodermic needle (H.N.) fitted

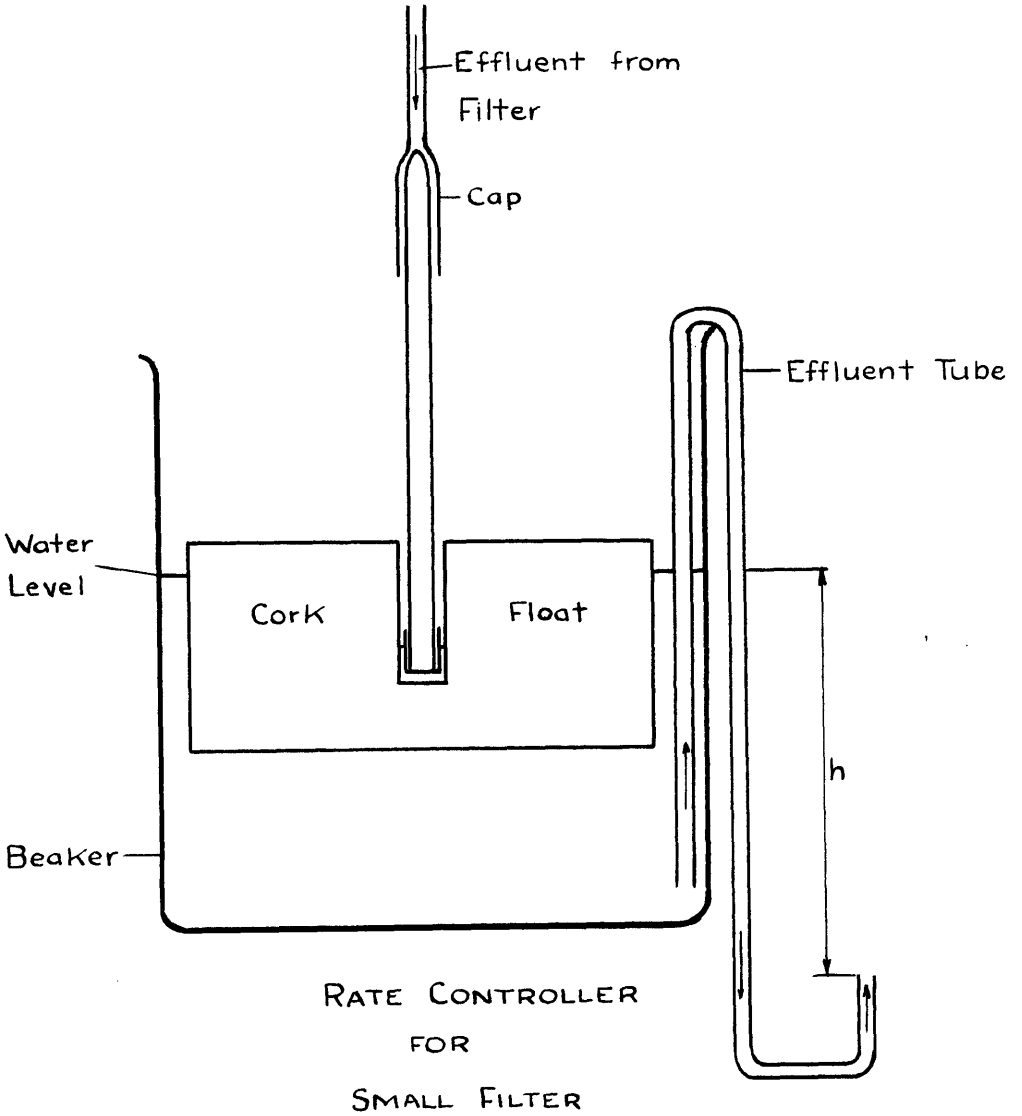
into the hole. At the other end of the needle, a rubber tube is connected which leads to a glass Tee (G.T.). One leg of the Tee is equipped with a pinch-cock and a short piece of tubing. This may be used as an air blow-off of for taking samples of the water. At the back leg of theTee, a rubber tube is attached which leads to the manometer board (M.B.). Thus, the pressure at each of these headers may be read in the manometer tubes at any instant during the run.

Water was supplied at constant pressure from the constant-level tank (T., Fig. 10). Through the pipe (P), the header (H) and the rubber tube (R.T.) it went to the filter. After passing through the sand, it went through the valve (E.V., Fig. 8) to the rate controller (R.C., Fig. 10). A diagram of this rate controller is given in Figure 11.

Attached to the cork float is a glass rod whose point is ground to fit the cap-shaped end of the tube through which the water comes from the filter. The opening between the end of this tube and the point of the rod governs the rate of flow. This opening is controlled through the action of the float by the level of water in the beaker. The rate of flow out of the beaker is controlled by the head (h), the lost head in the effluent tube. The head (h) is adjusted by raising or lowering the cap above the rod, thus changing the level of the float and the level of the water in the beaker.

The wash water arrangement may readily be seen in Figure 8. The rate of wash may be measured by the wash water manometer (W.M.).

Water enters the header (H) through the valve (W.V.). From the header, the water goes through the valve (V), through the gravel (G), which distributes the wash water, through the sand bed and through the header above the filter to waste.



RATE CONTROLLER
FOR
SMALL FILTER

FIG. 11

7. EXPERIMENTAL PROCEDURE

It was originally intended that all the experiments should be carried out with the small filter. Readings of the pressure at each of the headers were to be taken at frequent intervals to determine the lost head in various layers of the filter throughout the length of the run. About twelve times during each run, spaced at regular intervals throughout the run, samples of the water were to be taken at various depths in the filter and these samples were to be analyzed for the iron content of the water. From the values of iron content, the removal in each layer of the filter could be determined for each instant during the run and by a process of integration, the total iron removed could be obtained. The method for doing this is explained in detail on page 86. Knowing the ratio of iron content to the dry weight of suspended solids in the water, the total deposit of suspended solids in the filter could be determined. As a check on this result, it was planned to weigh the deposit in each small unit of the filter at the end of the run.

This filter was expressly designed for the purpose of weighing the deposit in certain units. These small units, 1½ inches deep, are of such a weight that with the sand in them, they may be weighed with accuracy on a sensitive chemical balance. Other units, of greater depth, were used in portions of the bed in which the weight of deposited matter was not desired. Screens at the top and bottom of the unit kept the sand grains enclosed so that the same grains were in the unit at the beginning and ending of a run. Before the run, it was planned to put the unit containing clean sand in a drying oven, and when dry and cool, to weigh the unit with its enclosed sand grains. The units would

then be put together to form the filter, taking care to expel all the entrapped air. The filter would then be operated as described on the preceding page. At the end of the run, the units would again be separated, dried in the oven, cooled and weighed. The increase in the weight of the unit would be equal to the dry weight of the matter deposited in that part of the filter during the run. As stated before, this weight should check with that calculated from the iron tests.

Experiments were begun with this filter, taking pressure readings every three hours, or around 40 readings per run of 120 hours. Samples of water were taken at various rates from the filter, but this disturbed the bed very markedly, as evidenced by a sharp change in the lost head in the filter. Different rates of sampling were tried, from as high as 40 c.c. per minute to as low as 10 c.c. per minute, and all disturbed the bed greatly. During the period of sampling, the rate of filtration through the sand in the top unit was made the same as that during regular operation, namely, 74 c.c. per minute, or 2 gallons per square foot per minute. This was done by adjusting the rate controller to give a rate equal to the difference between 74 c.c. per minute and the rate of sampling. Thus, if the rate of sampling were 10 c.c. per minute, the rate through the lower portion of the bed and through the rate controller was made equal to 64 c.c. per minute. This meant that the rate of filtration through the sand above the point of sampling was equal to 74 c.c. per minute. Sampling at a rate of 10 c.c. per minute was also tried with the rate controller untouched and the rate through the sand above the sampling point equal to the rate of sampling plus the rate of filtration, a total rate of 84 c.c. per minute. It was found that this method disturbed the bed also. At all the rates of sampling which were

tried, disturbances of the bed were noted, which made sampling impossible. Since samples of the water for iron tests could not be obtained from the small filter, the large filter had to be used for obtaining samples. Its area was so large that sampling did not disturb the bed.

However, the small filter was run in conjunction with the large filter. Equivalent amounts of sand were placed in each filter in order that they might be run in parallel. The results of lost head obtained in this filter were used as a check on those in the large filter. Another reason for running this small filter was to ascertain whether the two could be run in parallel. If weighing is to be done in the future, the small filter must be used for this purpose. Samples must be taken from the large filter. In order that the results of the weighing may be applied to the results obtained from calculations based on the iron tests of the samples in the large filter, the two filters must run in parallel, that is, the lost heads in the same layers of both filters must be equal, or very nearly so, and the iron removal in both filters must be the same. These experiments showed that the two filters could be run in parallel. The corresponding lost heads in both filters were almost the same and the total iron removal in the whole bed was the same for both filters throughout the length of the run.

For obtaining data in order to determine the shape factor of the sand, the small filter was used. There were more piezometer connections throughout the depth of this filter than there were in the larger one and these were all connected directly to manometer tubes so that the readings of the pressure during washing could all be taken simultaneously. The sand was washed at four different rates of flow

and readings of pressure and temperature were taken at each rate of flow. The rate was measured by collecting the quantity flowing for one minute and measuring this quantity in a graduate. This data was taken before the other experiments were begun in order to make sure that the sand was clean and that the bed contained no extraneous matter which would influence the results.

After the data for the shape factor were taken, the actual experiments on filtration were begun. The large filter was operated, pressure readings taken at frequent intervals, and samples taken every twelve hours, until the total loss of head in the filter reached a value somewhere between six and eight feet. The length of run for this purpose was usually from 70 to 140 hours.

Readings of pressure in the large filter were generally taken about every six hours, or from fifteen to twenty times during a run. Only one manometer tube on the manometer board was used, so that the rubber tube leading from this had to be connected to each piezometer tube on the filter successively. In order to release any air which might enter the tube during these changes, a glass Tee was inserted into the rubber tube connecting the piezometer and the manometer tubes. On the exposed branch a short piece of rubber tubing was connected, on which was placed a pinch-cock. By opening this pinch-cock and raising it above the general level of the rubber connecting tube, the entrapped air was expelled.

The temperature of the influent and effluent were read each time readings of the pressures were taken.

Approximately every twelve hours, or from eight to twelve times

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during a run, samples of water were taken through each of the piezometer tubes. The inlet to each tube was located in the center of the filter. During the run, the outlets of these tubes were sealed at the ends by a pinch-cock attached to a short piece of rubber tubing. A three-foot length of rubber tubing was used for sampling. At one end of this was a short piece of glass tubing. When sampling, this was inserted into the short piece of tubing at the end of the piezometer tube. At the other end of this three-foot tubing were a short hypodermic needle acting as a nozzle, and a pinch-cock just back of the nozzle, acting as a valve to regulate the rate of flow. These gave a fairly constant rate of flow. For five minutes the water was allowed to flow through the sampling tube at a rate of about 250 c.c. per minute. This was necessary to allow the coating on the sand grains in the immediate vicinity of the piezometer tube, where the rate of flow is much higher than in ordinary filtration, to wash off and flow to waste before taking the actual sample. After five minutes, the rate was cut down to around 100 c.c. per minute and allowed to flow for half a minute longer to be sure that all excess floc was washed out. Then a sample, between 250 and 500 c.c., was collected. This procedure was followed for each sample taken throughout the depth of the bed. The influent sample was taken out of the constant level tank, and the effluent sample from the effluent of the rate controller. Each sample was then tested for its iron content. Two or three times during a run, microscopic analyses were made to determine the average size of floc particles in these samples.

The filter was operated until the total head loss reached a value between 6 and 8 feet, giving a length of run between 70 and 140

hours. The sand bed was then washed. When beginning a wash, the wash water valve was opened very slowly in order to expand the bed evenly. The filter was washed at a rate which expanded the sand to a depth of about 160 percent of the depth during filtration until the water, after having passed through the sand, looked clean. Then the wash water valve was closed very slowly in order to permit the sand to settle to the same depth after each wash, and thus obtain the same average porosity at the beginning of each run.

The small filter was operated in a similar manner to the large one, except that no samples were taken during the run. Readings of the pressure were taken every three hours, or from twenty to forty times during a run. Each outlet in the filter was connected to a separate manometer tube on the manometer board so that the pressures could all be read simultaneously. This filter was washed immediately after the large one and in a similar manner.

8. RESULTS

The average sizes of floc particles at various depths for the different runs are given below in Table 1. Areas are all measured in standard units (1 standard unit equals 0.0004 sq. mm.). "Trace" signifies that a few small particles were visible, perhaps one or two in ten fields of 1 sq. mm. each. The hours represent the time at which the sample was taken, time being measured from the beginning of the run.

TABLE 1

Depth Feet	Run 6	Run 8		Run 9		Run 10		Run 12	
	Hrs. 90	47	95	21	45	15	63	22	118
Infl.	0.303	0.278	0.230	0.230	0.228	0.136	0.137	0.137	0.133
0.14	0.208	0.166	0.163	0.154	0.160	0.111		0.125	0.126
0.39	0.182	0.149	0.140	0.117	0.135	Trace		Trace	0.122
0.89	0.154	0.100	0.100	Trace	0.128				0.113
1.39	0.148	Trace	Trace		Trace				Trace
Effl.	0.100	Trace	Trace						Trace

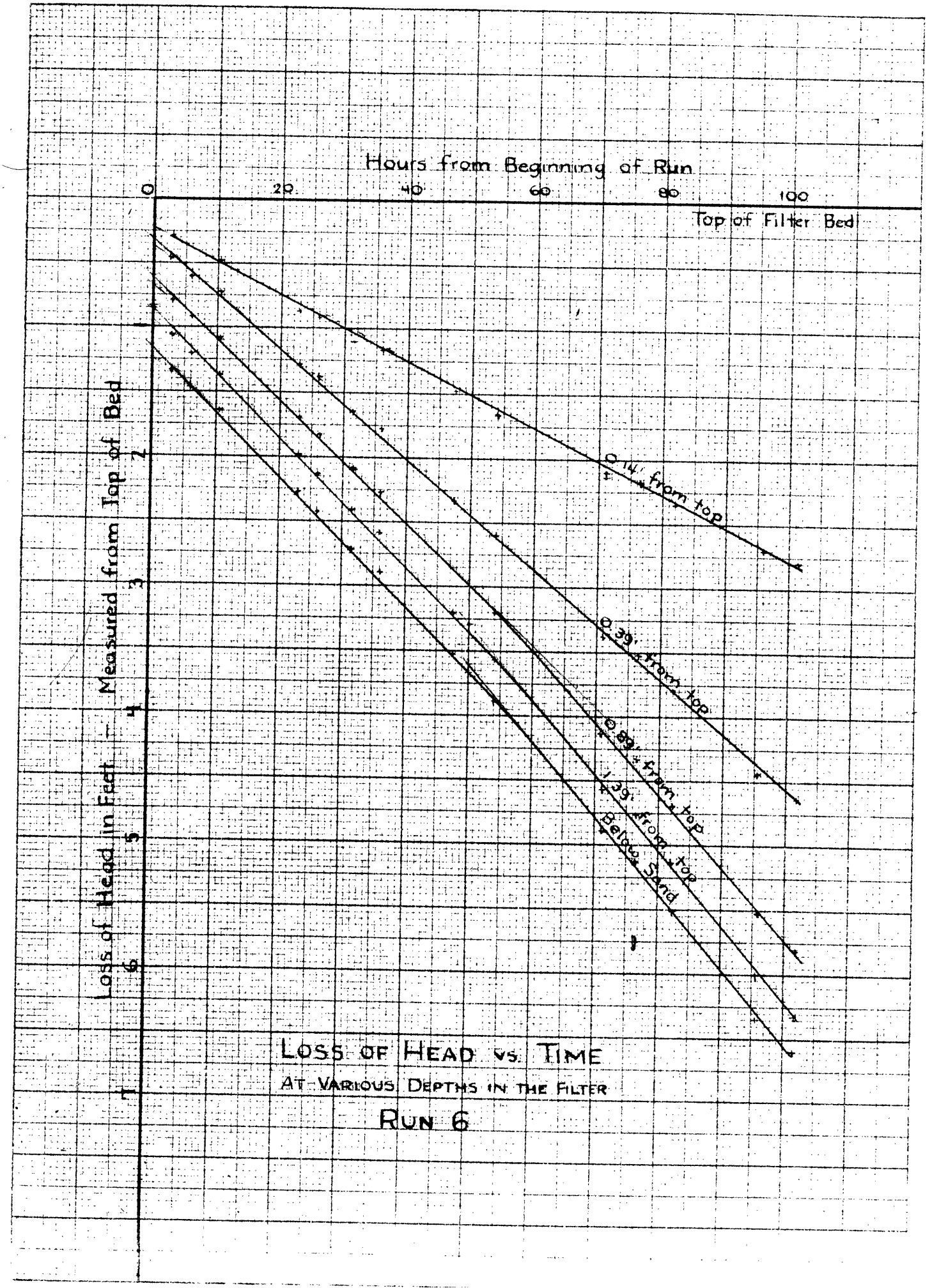
The results of these experiments are shown on four different kinds of graphs. One series (Fig. 12) shows the loss of head vs. time at various depths in the filter. The values of lost head have all been adjusted to the temperature of four degrees Centigrade, as explained on page 16, loss of head being proportional to coefficient of viscosity. The temperature did not vary much more than one degree either side of four degrees Centigrade, so the adjustments were very slight.

Another series of curves (Fig. 13) shows the iron content of the water vs. depth in the filter at various times during the run. Iron

content is measured in parts per million.

The third series of curves, (Fig. 14), is a combination of the two mentioned above. These curves show the removal in four succeeding layers of the filter. Iron removal and lost head in each layer are plotted as ordinates, with hours from the beginning of the run as abscissae. Iron removal is plotted as parts per million. These curves are plotted for the three runs in which the iron content of the water did not change during the run.

For Run 6 a curve of iron removal in parts per million per inch of depth is plotted against depth for every 20 hours during the run. (Fig. 25). The iron removal in each layer is divided by the depth, in inches, of the layer. This value is the average removal per inch of depth of the layer. Being the average, the value is plotted at the center of the layer, and the curves drawn accordingly. These curves give the iron removal per unit depth throughout the whole depth of the filter at various times during the length of the run.

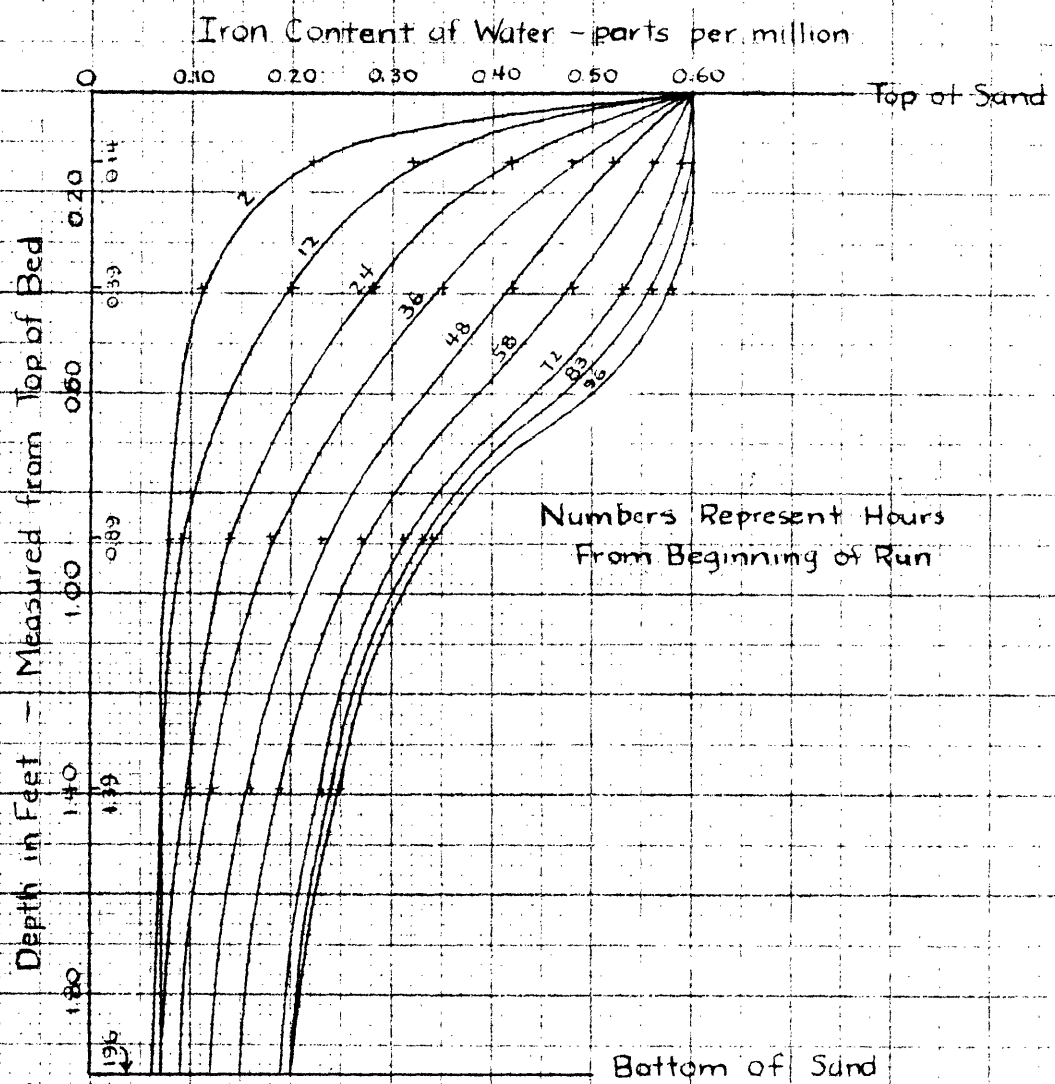


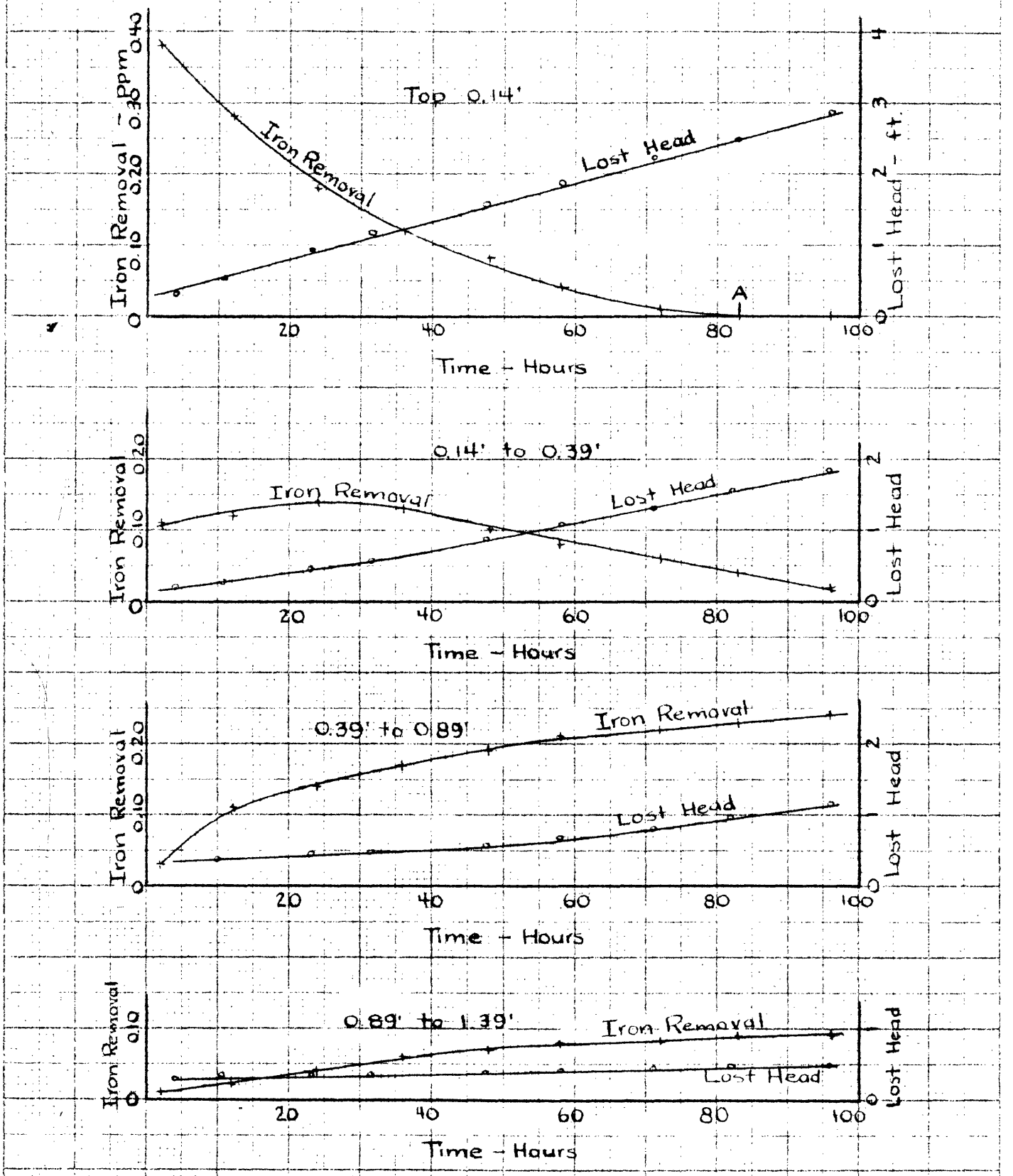
LOSS OF HEAD vs. TIME

AT VARIOUS DEPTHS IN THE FILTER

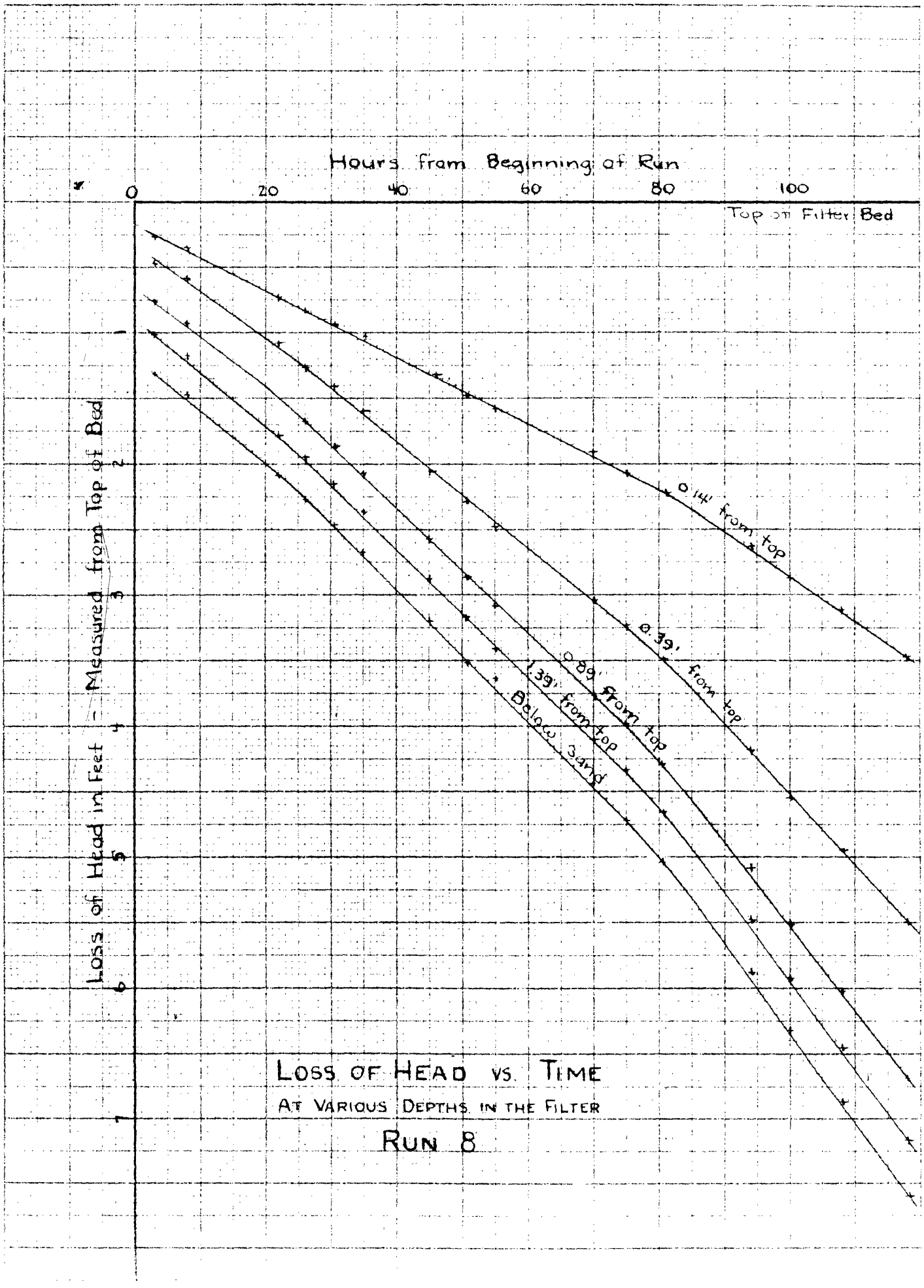
RUN 6

Iron Content vs. Depth in Filter
At Various Times During the Run
Run 6





IRON REMOVAL AND LOST HEAD
VS. TIME
FOR VARIOUS LAYERS OF THE FILTER
RUN 6

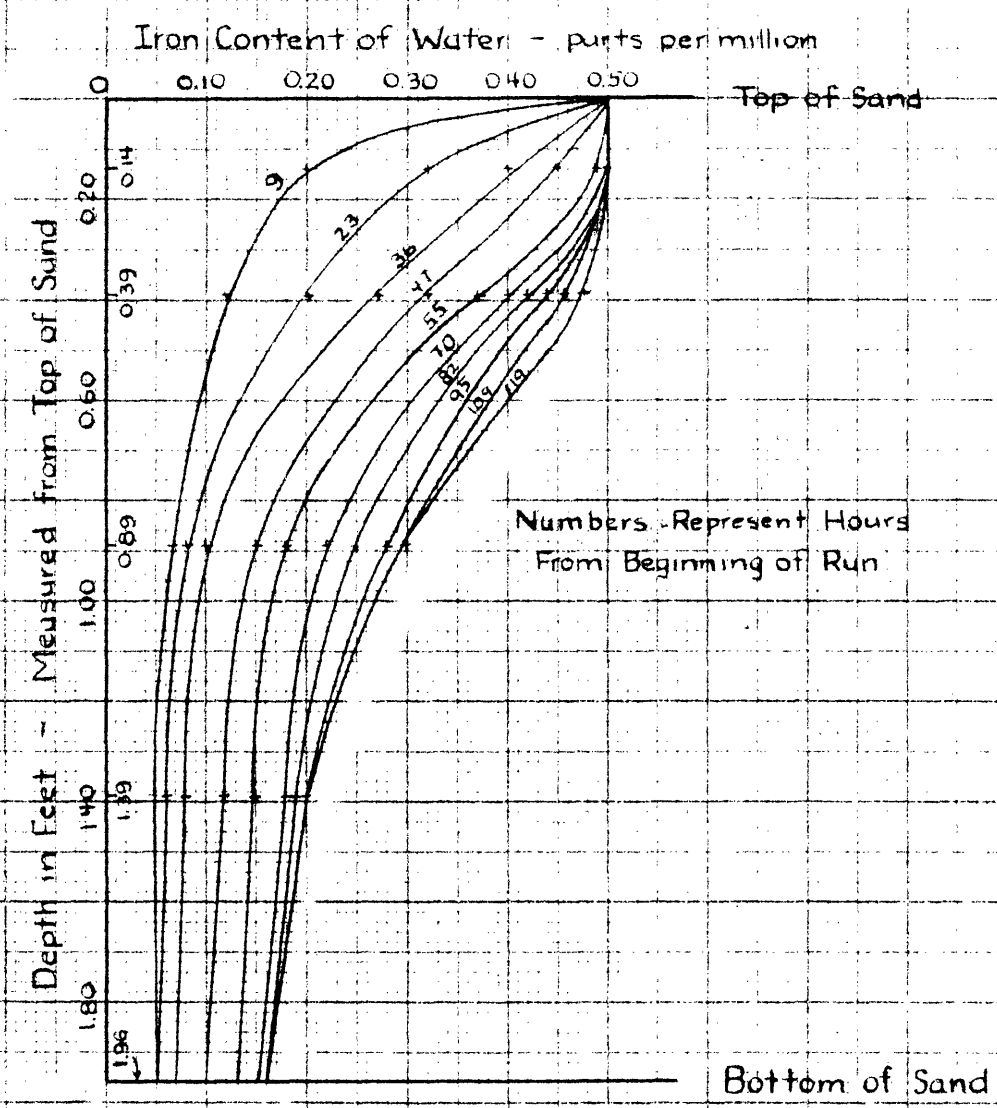


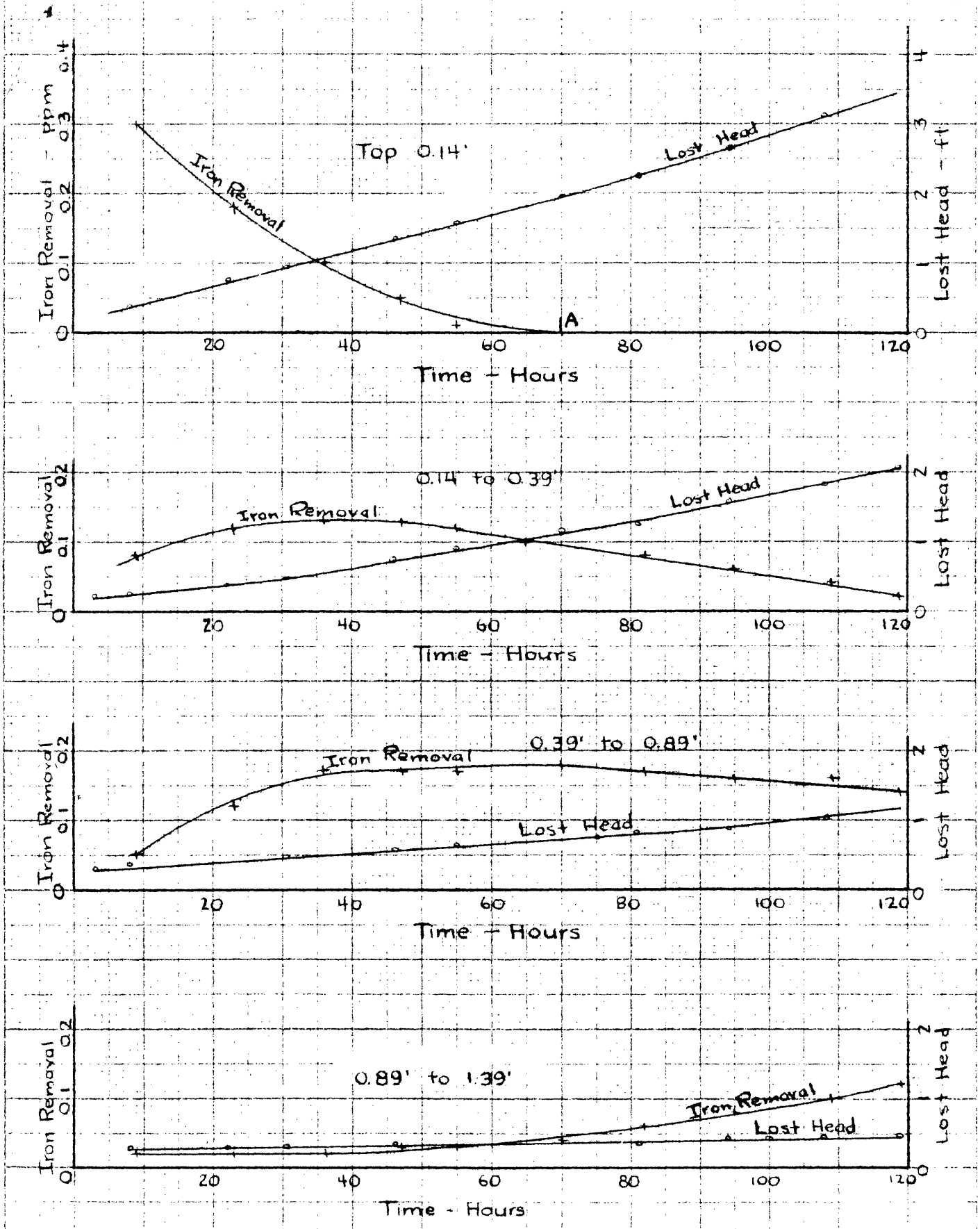
LOSS OF HEAD VS. TIME

AT VARIOUS DEPTHS IN THE FILTER

RUN 8

Iron Content vs. Depth in Filter
At Various Times During the Run
Run 8

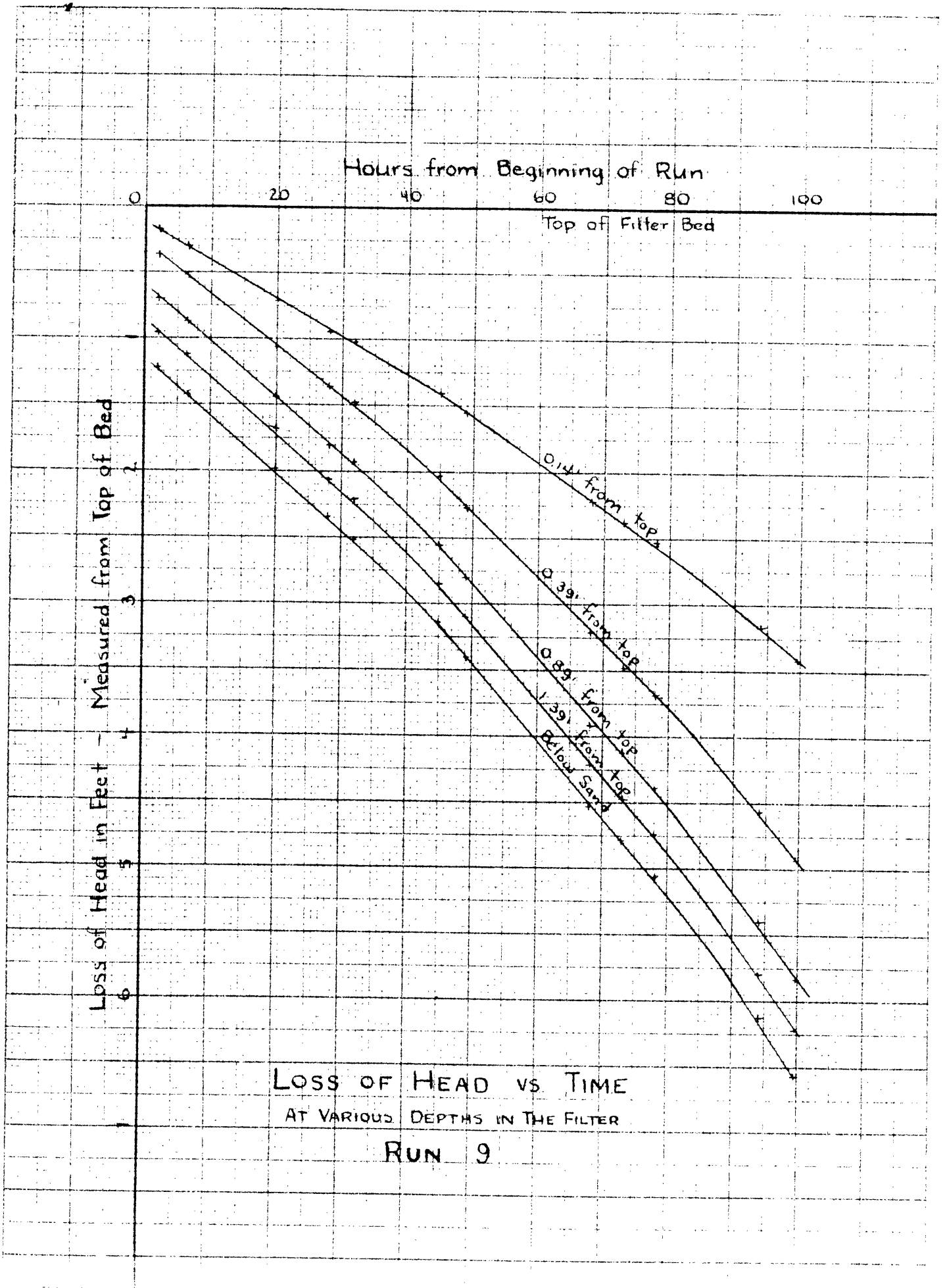




IRON REMOVAL AND LOST HEAD VS. TIME

FOR VARIOUS LAYERS OF THE FILTER

RUN 8



LOSS OF HEAD VS TIME

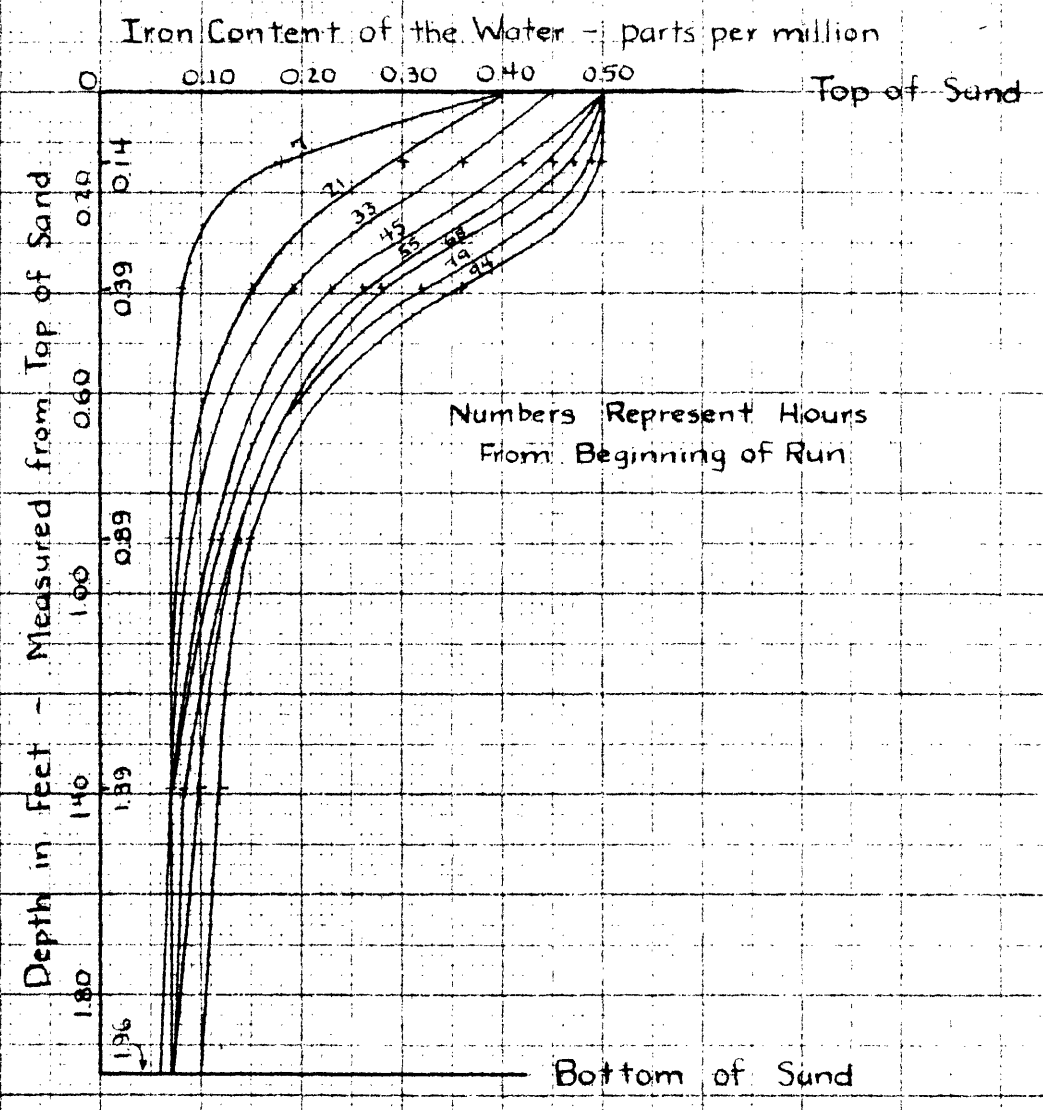
AT VARIOUS DEPTHS IN THE FILTER

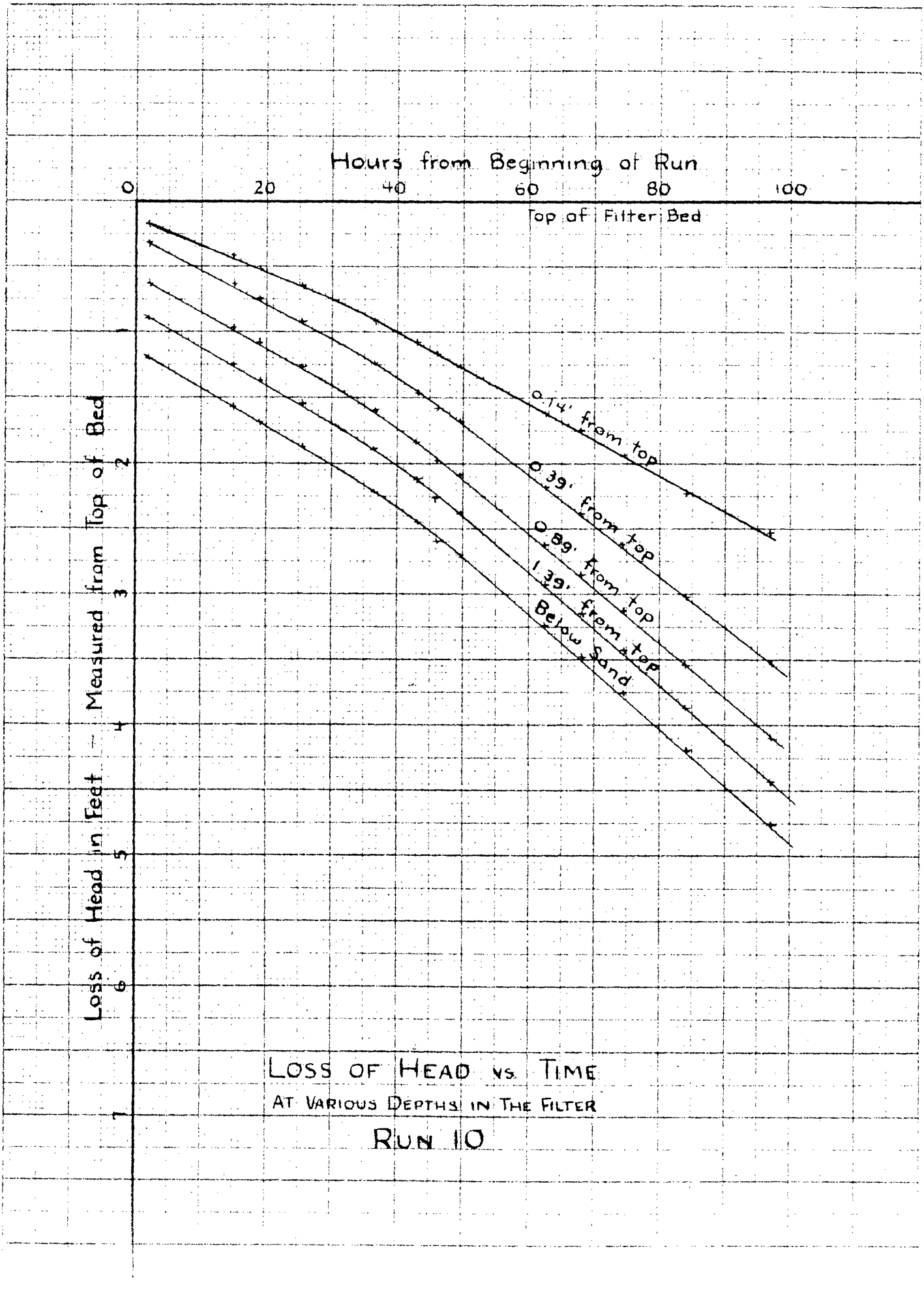
RUN 9

Iron Content vs Depth in Filter

At Various Times During the Run

Run 9





LOSS OF HEAD VS. TIME

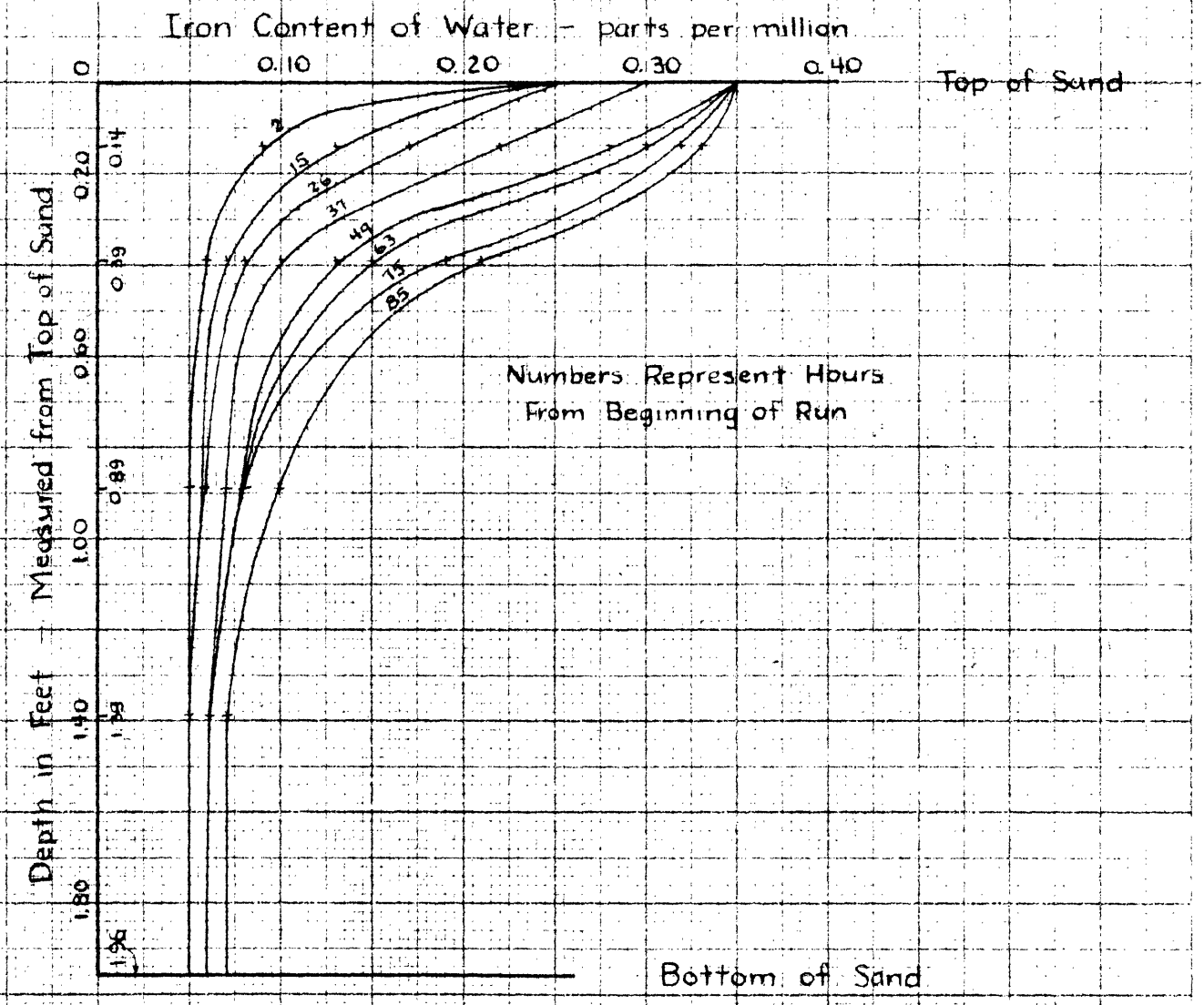
AT VARIOUS DEPTHS IN THE FILTER

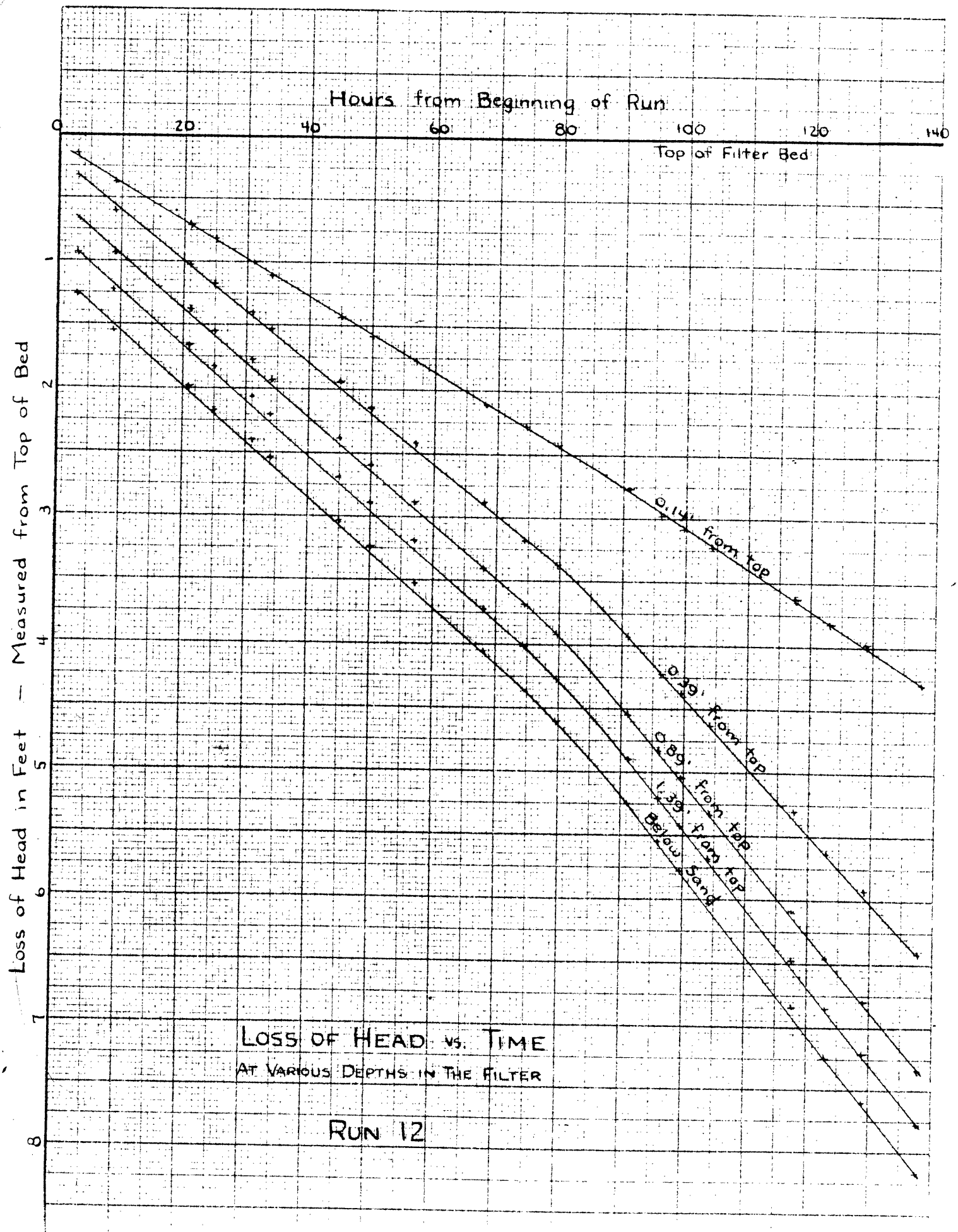
RUN 10

Iron Content vs Depth in Filter

At Various Times During the Run

Run 10

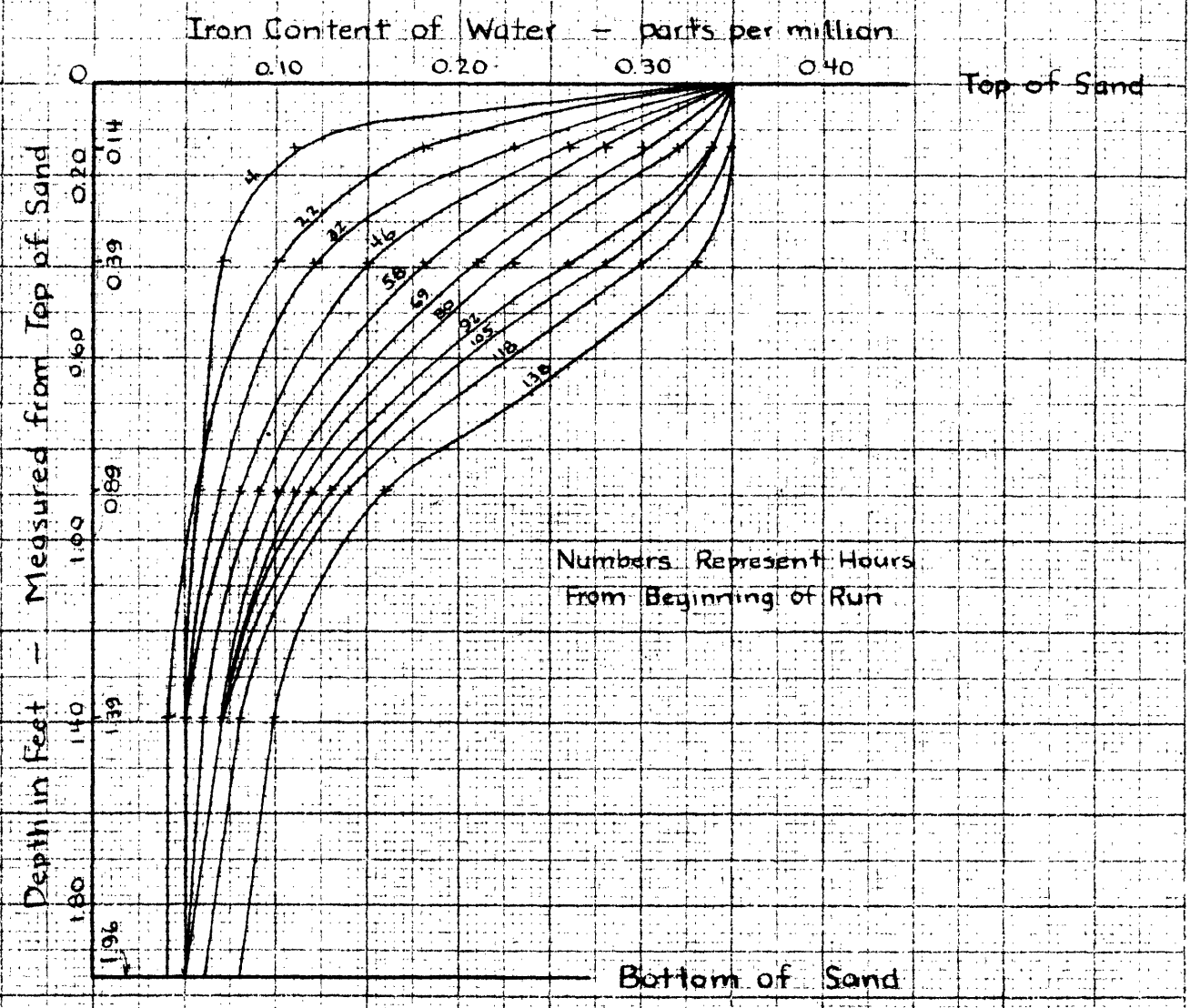


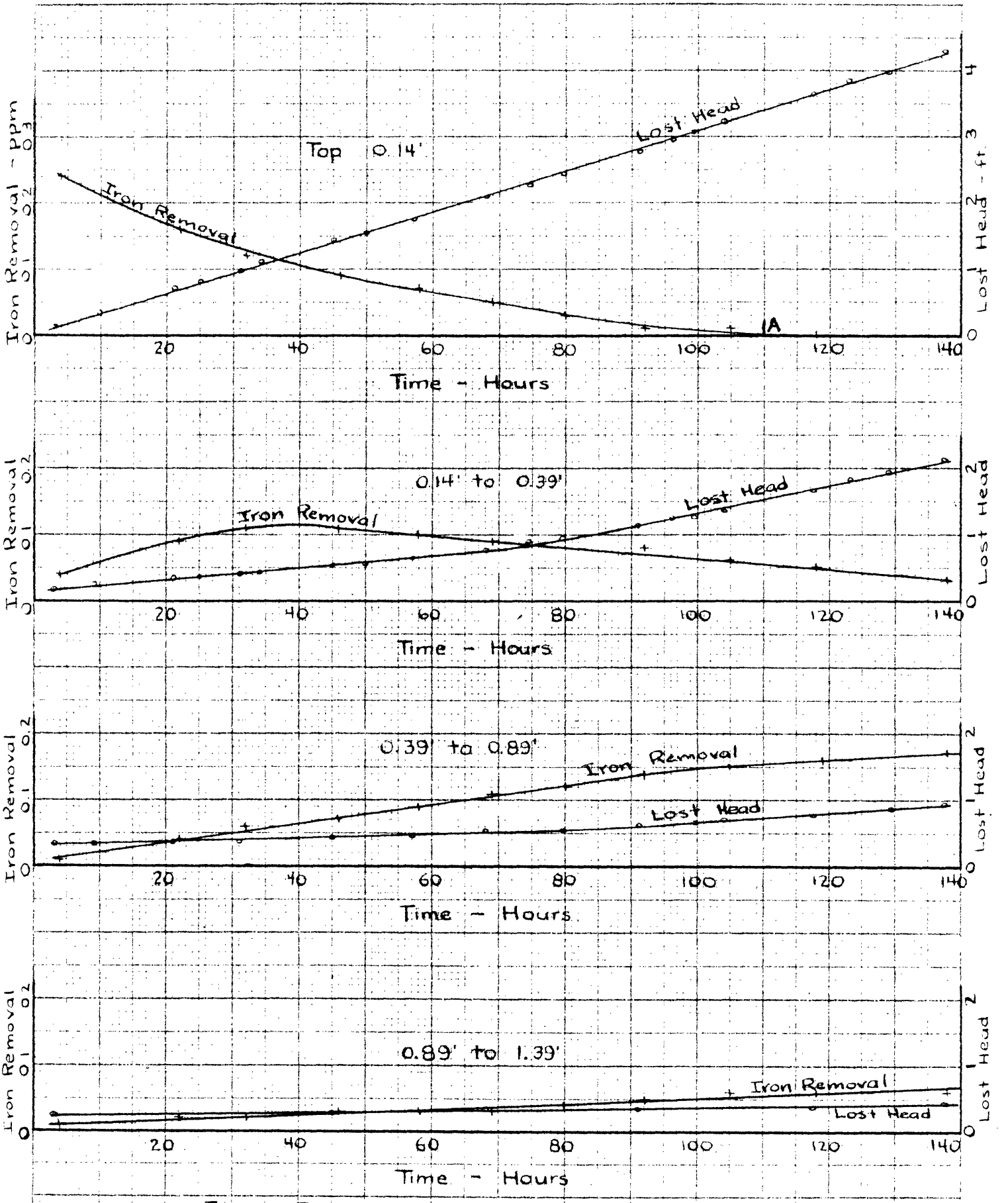

Hours from Beginning of Run
0
20
40
60
80
100
120
140
Top of Filter Bed
Loss of Head in Feet — Measured from Top of Bed
1
2
3
4
5
6
7
8
0.14' from top
0.39' from top
0.89' from top
1.39' from top
Below Sand
LOSS OF HEAD vs. TIME
AT VARIOUS DEPTHS IN THE FILTER
RUN 12

Iron Content vs. Depth in Filter

At Various Times During The Run

Run 12





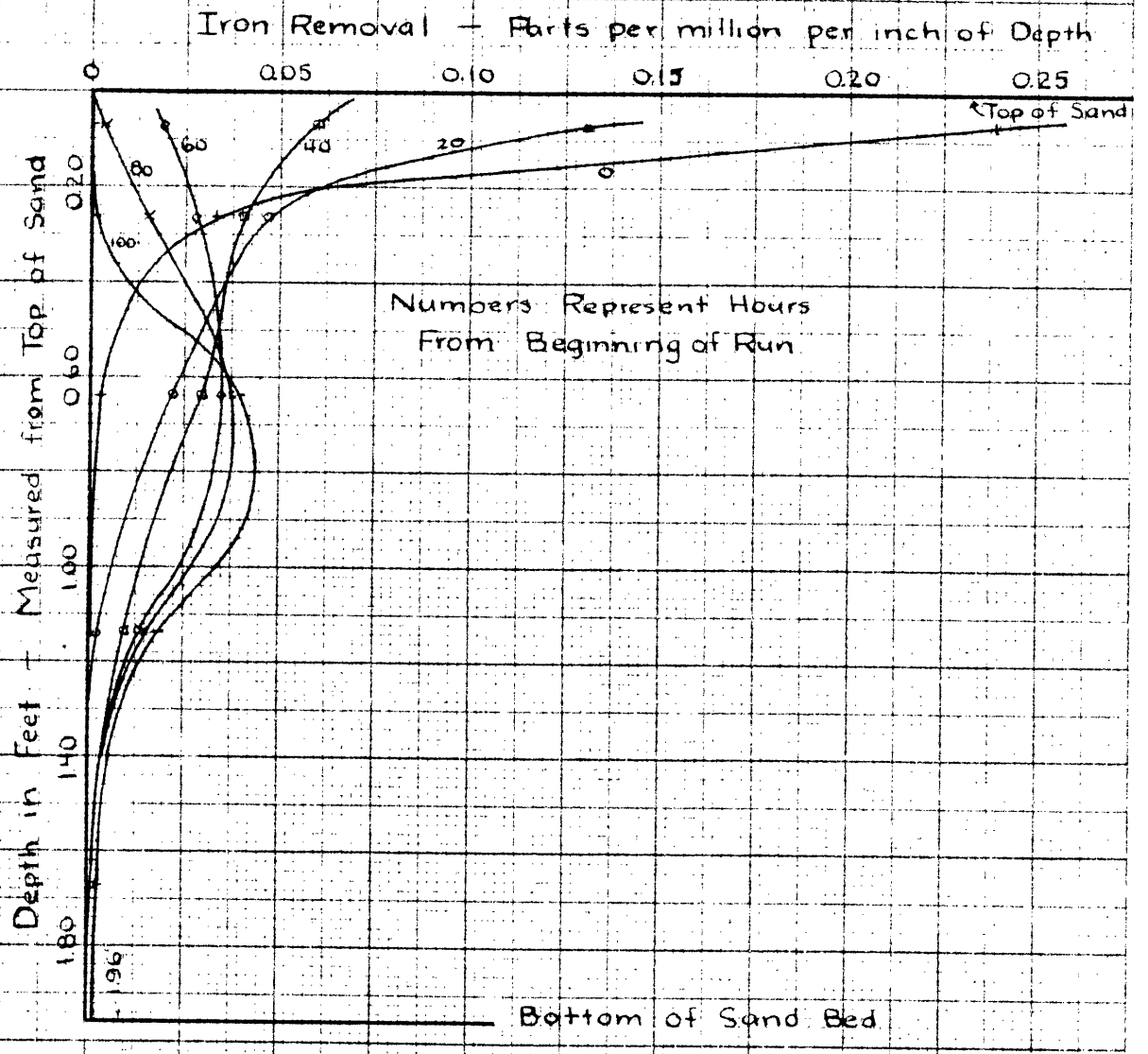
IRON REMOVAL AND LOST HEAD VS TIME

FOR VARIOUS LAYERS OF THE ALTER

RUN 12

Iron Removal per Inch of Depth vs Depth at Various Times During the Run

Run 6



9. DISCUSSION AND ANALYSIS OF RESULTS

Some of the most important results of this research are those which show, for the first time, the author believes, the time rate of removal of solid matter from the water in different layers of the filter, and the variation of this removal during the length of the run. The curves of iron removal and lost head vs. time (Figs. 14, 17, and 24) in various layers of the filter for Runs 6, 8, and 12 show how the rate of removal of solid matter from the water varies in each layer during the length of the run.

In the layer comprising the first 0.14 feet of depth in the filter, the iron removal, and with it, the removal of the colloidal and suspended solids comprising the floc particles, begins at its maximum value and continually decreases until a saturation point is approached. The discussion of the behavior of the iron removal curve as it approaches the line of zero removal will be referred to on a later page. Suffice it to say that although the curve shows zero removal, actually a very slight removal takes place, although so small that the tests for iron content were not able to detect it. The line of zero removal is really the asymptote to this curve.

The second layer is 0.25 feet in depth, extending from 0.14 to 0.39 feet below the top of the sand bed. The removal in this layer begins at a fairly high value and increases as the removal in the top layer decreases. A period of maximum removal is reached and the removal then decreases gradually until, at the end of the usual run--total lost head between 6 and 8 feet--the layer is nearly saturated with flocculated matter.

The third layer is 0.50 ft. in depth, and extends from 0.39 to 0.89 feet below the surface of the sand bed. Removal in this layer begins at a low value, since the two layers above are removing most of the flocculated matter. But as the removal in the top layer decreases rapidly, and the removal in the second layer reaches its maximum and begins to decrease, the removal in the third layer increases steadily and near the end of the run it carries most of the burden of removal.

The fourth layer in this filter is 0.50 feet deep and extends from 0.89 to 1.39 feet below the surface. During the first part of the run, very little iron is removed in this layer. As the two top layers decrease their removal, the third layer cannot remove all the flocculated matter which comes to it, and more passes through to this layer. Near the end of the run, it removes considerable iron from the water.

The final layer of the filter is 0.57 feet deep, extending from 1.39 feet to the bottom of the sand bed, 1.96 feet from the surface. The iron removal and the increase of lost head during the run in this layer are practically negligible.

The curves of iron removal during Run 6 per unit depth vs. depth in the filter (Fig. 25) show how the removal is a maximum in the top portion of the bed for a considerable part of the run, but the section of maximum removal moves downward as the upper pores become clogged.

The depth to which floc penetrates in the filter has long been a topic for discussion. The curves of iron content vs. depth in the

filter (Figs. 13, 16, 19, 21 and 23) show that floc penetrates through the bed all the time, since iron may always be found in the effluent.

Others may consider the depth of penetration as the lowest depth at which they can see floc with the naked eye. Armstrong determined the depth of penetration by collecting samples of the sand grains and deposit at different depths. Each sample was placed in a beaker and the sediment thoroughly washed from the sand. The sediment was collected on filter paper, burned and later weighed. As a result of his experiments he concluded as follows: "Even after long filter runs, the floc does not penetrate to a depth of over 2 or 3 inches for the fine sands, and that the penetration gradually increases until in the coarse grained filters the floc permeates the entire sand bed in a single long run."[1]

It is true that in the experiments described herein, there is a depth beyond which the removal is slight. The curves of iron content vs. depth in the filter for Run 8 (Fig. 16) show that at the beginning of the run, the removal is slight below a depth of 1 foot. As the filter clogs, considerable removal takes place at this depth, and the depth at which removal is slight advances downward until, at the end of the run, even the bottom layer takes part in the removal. The fact that floc penetrates the whole depth of the bed is further shown by the fact that after a series of runs, the sand has been colored considerably, floc particles being adsorbed on the surfaces of the sand grains.

In looking at the curve of iron removal vs. time for the first layer of Run 6 (Fig. 14), Run 8 (Fig. 17), Run 12 (Fig. 24), it will be noticed that the iron removal in this layer apparently ceases before the end of the run is reached. Thus, it would appear that the layer

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1. Armstrong, J. W., Journ. Am. Water Works Assn. 23: p. 1299, Sept.1931

has reached a saturation point at which no more floc may be deposited. Still, the lost head continues to build up at practically a constant rate, regardless of this saturation. This would tend to indicate that flocculated matter is still being removed from the water in passing through this layer. Of course, consolidation of the bed may cause some increase in lost head, but this alone would not keep the time rate of increase of lost head so nearly constant. It is probable that the test for the iron content of the water is not sensitive enough to indicate the slight amount of iron which may be removed in this top layer.

The equation of filtration (Equation 1, p. 15) will be used to analyze what is happening as the removal in the layer approaches zero. This equation reads:

$$\frac{h}{l} = \frac{K\mu}{w} v \frac{(1-f)^2}{f^3} \frac{S^2}{d^2} \qquad (1)$$

The possible variations of the factors of this equation were discussed in a previous section (pp. 15 to 21). The small amount of deposit at the apparent saturation point (point A, Fig. 14), as indicated by the immeasurable amount of iron removed, will have a negligible effect on K, S, and d, since the sand grains are quite heavily coated already. But the effect of this small amount of deposit on the porosity will be considerable, because the porosity is already a small value when the top layer is near its saturation point. When the porosity is quite low, a small change in porosity will have a large effect on the lost head, since h is proportional to $\frac{(1-f)^2}{f^3}$. In Fig. 26, the porosities are plotted as ordinates to a natural scale, and the corresponding

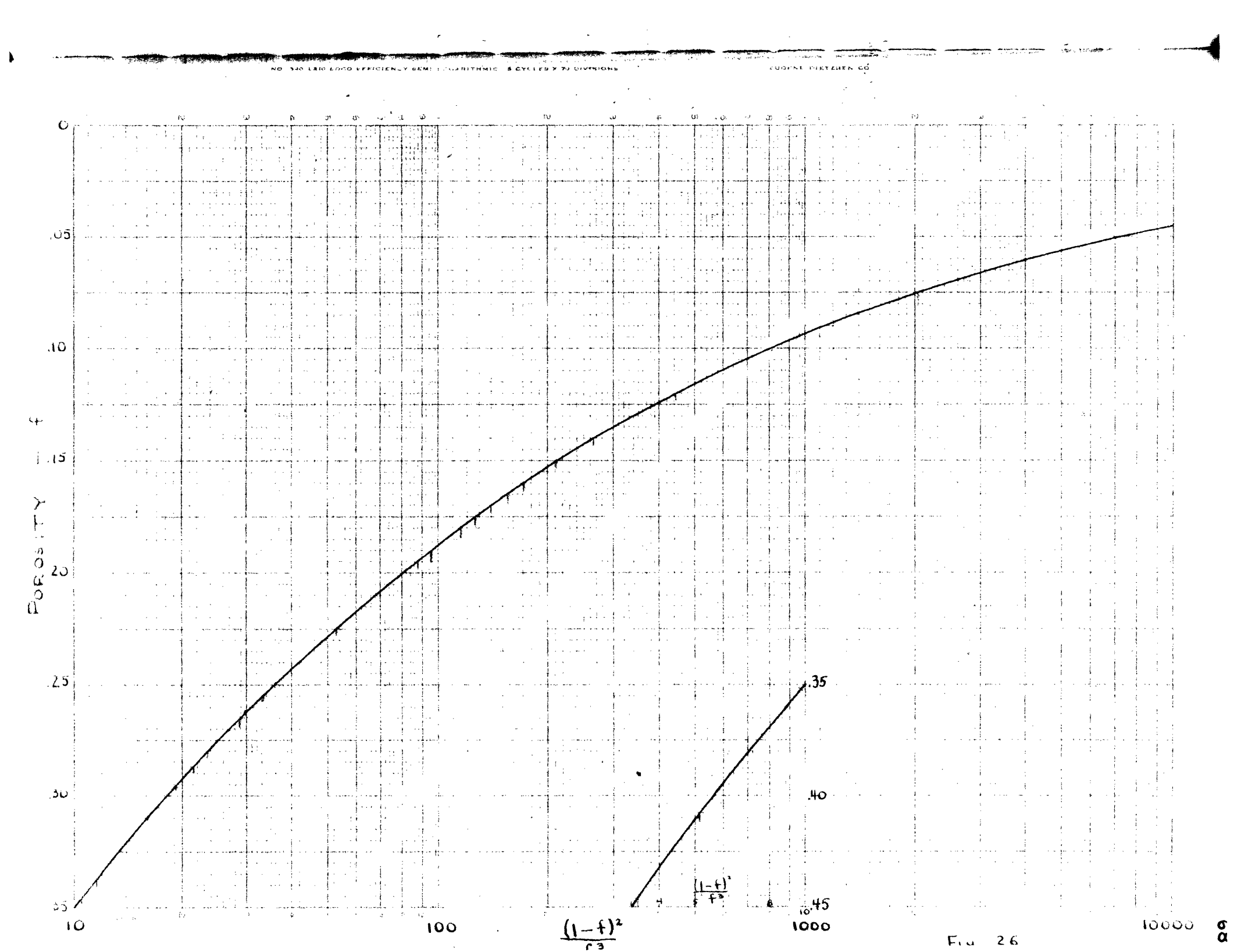

Porosity f
0
.05
.10
.15
.20
.25
.30
.35
.40
.45
10
100
1000
10000
$\frac{(1-f)^2}{f^3}$
$\frac{(1-f)^2}{f^3}$

Fig. 26

σ
α

values of $\frac{(1-f)^2}{f^3}$ are plotted as abscissae to a logarithmic scale. From this curve, it may be seen that $\frac{(1-f)^2}{f^3}$ increases at a steadily increasing rate. Thus, for a unit decrease of porosity when the porosity is 40 percent, $\frac{(1-f)^2}{f^3}$ increases from 5.6 to 6.2, an increase of 11 percent. For a unit decrease of porosity when the porosity is 10 percent, $\frac{(1-f)^2}{f^3}$ increases from 800 to 1120, an increases of 40 percent. Thus, it may be seen that a small decrease of porosity, when the porosity is already low, will have a marked effect on the increase of lost head. This seems to indicate that the iron removal in the layer does not actually reach a saturation point, but is such a small value that the iron tests cannot detect it. It appears that the curve becomes asymptotic to the zero-removal axis.

At point A (Fig. 14), where the iron removal curve becomes asymptotic, the changes in K,S, and d are negligible, and f is the only variable. Therefore, Equation 1 may be written in the following manner:

$$h = K_1 \frac{(1-f)^2}{f^3} \qquad (4)$$

where $$K_1 = \frac{K\mu}{w} v l \frac{S^2}{d^2} \qquad (5)$$

In Equation 4 the only variables are h and f. The values of h are known from experiment. Therefore, if K_1 is known, the values of f may be obtained. Having the value of f, it is possible to obtain the value of the volume in the filter occupied by the deposited matter. This volume may be compared with the mass curve of the total deposit of iron in the layer since the beginning of the run. By

adjusting the scales, the curve of volume of deposit obtained from the calculated porosity may be compared with the mass curve. The object of this comparison of curves is to see how the slopes of the two curves compare at time A (Fig. 14). A coincidence of the two curves will indicate approximately the rate of deposit which is taking place at Time A.

The procedure outlined above will now be described in detail for Run 8. As a first assumption, the value of K_1 will be chosen to be the same as it was at the beginning of the run in the first layer. At the beginning of the run, Fig. 17 shows the lost head equal to 0.15 feet. The initial porosity equals 40.8 percent. From Fig. 26, the value of 5.5 is read for $\frac{(1-f)^2}{f^3}$.

$$H = \frac{(1-f)^2}{f^3} \cdot K_1 \qquad 0.15 = K_1 \times 5.5$$

$$K_1 = 0.0272$$

For any other lost head, H, $\frac{(1-f)^2}{f^3} = \frac{H}{K_1}$. At time A (Fig. 17), the lost head equals 1.95 feet. Dividing 1.95 by K_1, a value of $\frac{(1-f)^2}{f^3}$ of 71.7 is obtained. In Fig. 26, this corresponds to a porosity of 20.6 percent. Subtracting this from the initial porosity, 40.8, 20.2 percent of the total volume of the filter is obtained as the volume of deposited matter in the layer at time A. This assumes that K_1 has the same value that it had at the beginning of the run. In like manner, the values of the percent of volume of deposited matter at ten-hour intervals for Runs 6, 8, and 12 may be computed. These values are shown plotted against time in Fig. 27.

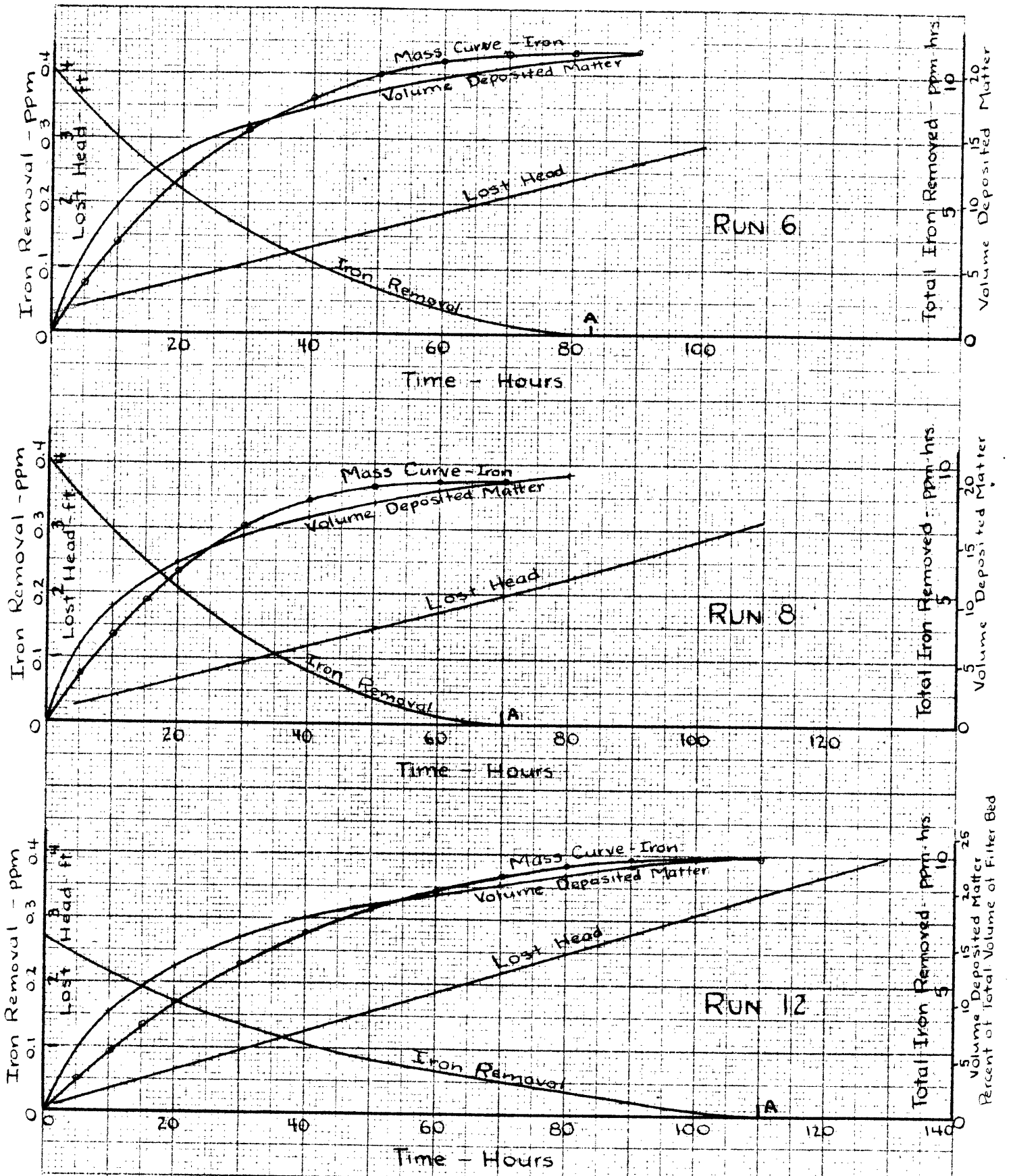
The mass curve of the total quantity of iron removed in the first

layer since the beginning of the run may be obtained by integrating the curve of iron removal (Fig. 17). This is done by finding the area under the curve for every 10 hours of the run and adding each area to the sum of the preceding areas. The units of these areas will be parts per millionxhours. Using certain conversion factors, the equivalent amount of iron may be obtained. This is worked out in detail on page 87. For the purpose of comparing curves, the units of parts per million x hours will suffice. The mass curves for Runs 6, 8 and 12 are shown in Fig. 27.

Fig. 27 also shows the lost head and the iron removal in the top layer for Runs 6, 8, and 12. These curves were taken from Fig. 14, 17 and 24. The scale to which the volume of deposited matter, expressed as percent of the total volume of the filter, is plotted, has been selected so that the two curves coincide at time A and also have the same origin.

From these curves, it may be seen that the slope of the computed curves of volume of deposited matter is too great at time A. These curves should coincide with the mass curve which has been determined from the results of the iron tests. It appears that the assumptions made in computing these curves are false. Therefore K_1 is not constant throughout the run. In other words, the product $K\frac{S^2}{d^2}$ must vary during the run. The steep slope of these computed curves also shows that the computed porosities, around 20 percent, are too high.

Other assumptions will now be made in an effort to get computed curves which will correspond with the actual mass curves. An assumption will be made for the porosity in the top layer at time A of Run 6. The



TOTAL IRON REMOVED - IRON REMOVAL
VOLUME DEPOSITED MATTER - LOST HEAD
VS TIME
FOR THE TOP LAYER IN THE FILTER

FIG. 27

values of the volume of deposited matter, expressed as percent of the total volume of the filter, may be computed in a manner similar to that used on page 70, using a value of K_1 corresponding to the porosity assumed at time A. The slope of this curve may be compared with that for the mass curve, and assumptions made for the porosity until a porosity is found which will give a computed curve having a slope, just before time A, equal to the slope of the mass curve.

Run 6 will be used. Assuming a porosity at time A equal to 15 percent, $\frac{(1-f)^2}{f^3}$ equals 212 (Fig. 26). The lost head at time A equals 2.40 feet (Fig. 12). K_1 equals 2.40 divided by 212, or 0.0113. At 80 hours the value of the lost head is 2.28 feet. Dividing this by K_1 (2.28 divided by 0.0113), the value of $\frac{(1-f)^2}{f^3}$ equal to 202 is obtained. From Fig. 26 the value of the porosity is seen to be 15.2 percent. In like manner, the porosity at 70 hours may be computed, a value of 15.8 percent resulting. The volume of deposited matter, expressed as percent of the total volume of the filter,(hereafter designated as V.D.M) at 85 hours equals 40.8 - 15.0 or 25.8. At 80 hours V.D.M. equals 25.6, and at 70 hours, 25.0. The ordinates to the mass curve are respectively 10.8, 10.8 and 10.75. Reducing the V.D.M.'s to a scale similar to the mass curve values is done by making the values for the two curves at 85 hours both equal to the value of the ordinate to the mass curve and reducing the others in proportion. Thus the V.D.M. at 85 hours is made equal to 10.8. The value of the V.D.M. at 80 hours equals $\frac{10.8}{25.8}$ x 25.6, or 10.7. Similarly the value for 70 hours equals 10.45. These values are tabulated below:

TABLE 2

Porosity at A 15 percent

Hours	Mass Curve	V.D.M.
85	10.80	10.80
80	10.80	10.70
70	10.75	10.45

It is desired to have the slopes of the two curves identical, or nearly so, as the mass curve approaches time A. The slope of the mass curve between 70 and 80 hours, just before the curve reaches the apparent saturation point (Time A), will be chosen as that slope to which the slope of the computed curve must be equal. Between these hours, the sand grains may be considered so heavily coated that K, S, and d do not vary, and that porosity is the only variable. Thus, with the porosity at A assumed at 15 percent, the value of the slope of the V.D.M. curve is $\frac{10.70 - 10.45}{10}$, or 0.025. The value of the slope of the mass curve between these hours is $\frac{10.80 - 10.75}{10}$, or 0.005. This difference of slopes tends to indicate that the porosity assumed is too high.

A porosity of 10 percent at time A for the top layer in Run 6 will be tried now. Computations are made in a manner similar to those above. The results are as follows:

TABLE 3

Porosity at A 10 percent

Hours	Mass Curve	V.D.M.
85	10.80	10.80
80	10.80	10.75
70	10.75	10.60

The slope of the V.D.M. curve is 0.015 while that of the mass curve is 0.005. This difference seems to indicate that 10 percent is too high for the porosity at Time A.

The following are the results obtained assuming a porosity of 5 percent at time A:

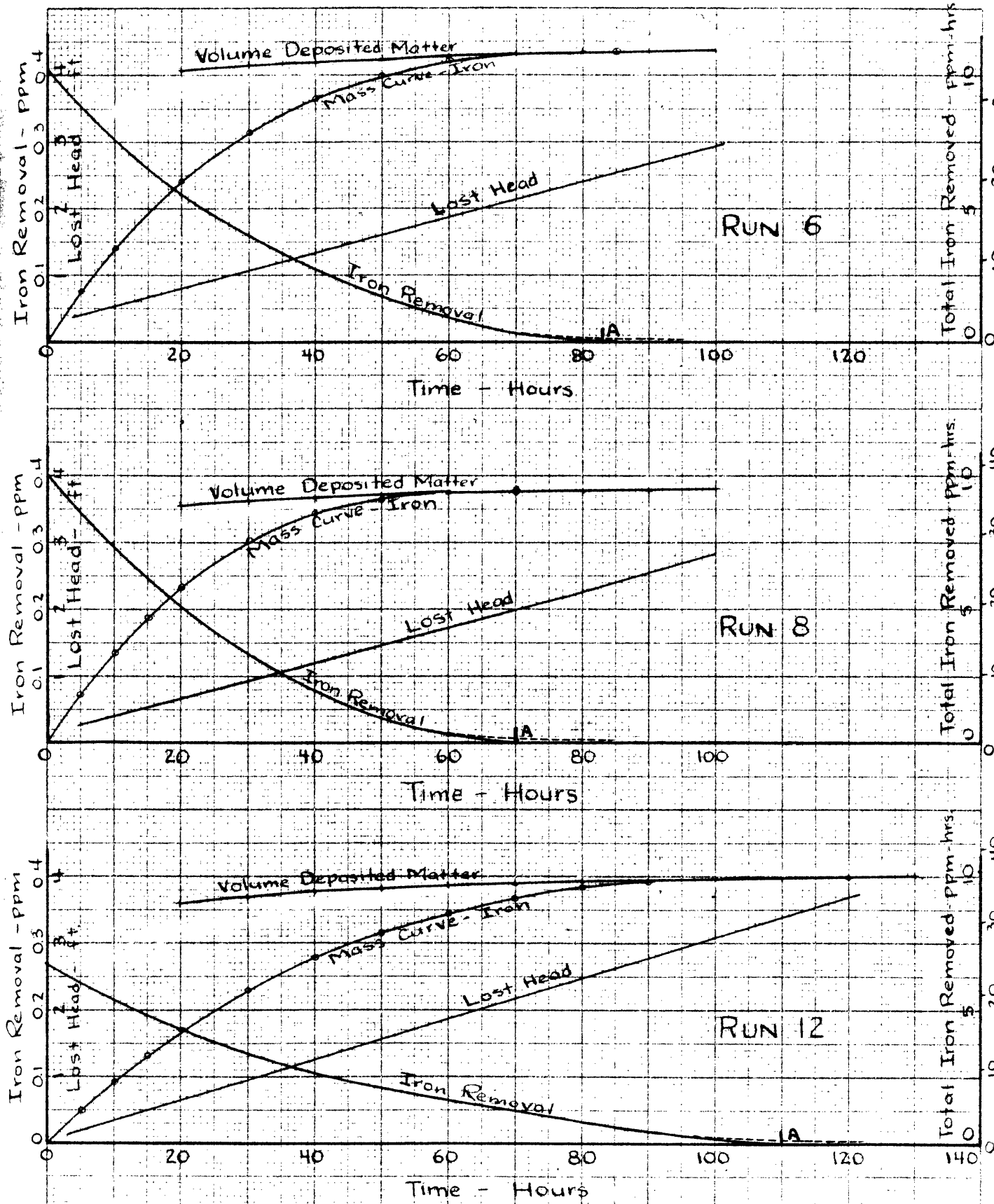
TABLE 4

Porosity at A 5 percent

Hours	Mass Curve	V.D.M.
85	10.80	10.80
80	10.80	10.775
70	10.75	10.725

The slope of the V.D.M. curve is 0.005 and the slope of the mass curve is also equal to 0.005. This identity of slopes seems to indicate that a porosity of 5 percent is within a few percent of being the correct porosity at the time the iron removal curve in the first layer becomes asymptotic to the zero-removal axis.

Assuming that the porosity at time A in Runs 6, 8 and 12 is equal to 5 percent, the value of K_1 for each of these runs is computed. The value of K_1 for Run 6, assuming the porosity at time A to be 5 percent, is equal to the value of the lost head, 2.40, divided by 7400, the value of $\frac{(1-f)^2}{f^3}$ when f = 5 percent. K_1 equals 0.000324. K_1 for Runs 8 and 12 is obtained in like manner. The values of the V.D.M.'s are computed for every ten hours during each run in a similar manner to that described on p. 70. These values are then plotted and the curves shown in Fig. 28 are drawn. These curves of the volume of deposited matter (V.D.M.), expressed as percent of the total volume of



TOTAL IRON REMOVED - IRON REMOVAL
VOLUME DEPOSITED MATTER - LOST HEAD
VS. TIME
FOR THE TOP LAYER IN THE FILTER

FIG 28

the filter, are based on the assumption that the value of the porosity in the top layer at time A is equal to 5 percent. The other curves shown in Fig. 28 are exactly like those in Fig. 27.

The coincidence of the mass curve and the computed curve of the volume of deposited matter proves that the mass curve should not stop at time A, but should continue at the same flat slope that the computed volume of deposit curve has. This means that the curve of iron removal for the top layer should not actually become tangent to the zero-removal axis. This curve should be asymptotic to the axis, which means that a saturation point is not actually reached during the run. A very slight removal takes place, so slight that the test for iron cannot detect any removal in the layer. This slight removal is indicated by the very flat slope of the computed curve of the volume of deposited matter.

The coincidence of these curves also shows that porosity is by far the main factor influencing the lost head in the top layer during that part of the run in which the iron removal curve is asymptotic to the zero-removal axis.

The divergence of these curves is caused by the other factors which influence the lost head during the earlier portions of the run, namely, K, S, d, and the consolidation of the bed. A measure of the magnitude of the total change in these factors may be obtained by comparing the values of K_1 at the beginning of the run and at time A. K_1 is equal to $K \frac{S}{d^2} \left(\frac{\mu v l}{w}\right)$, the factor in the parentheses remaining constant throughout the run. K_1 at the beginning of the run was found to be equal to 0.0272 (p.70). At time A, assuming a porosity of 5 percent, K_1 was found to be equal to 0.000324. This means that K_1 is 84 times

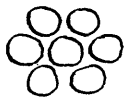
larger at the beginning of the run than at time A.

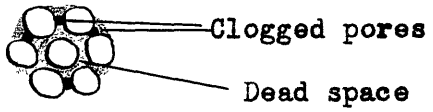
There are two ways in which the deposit in the bed may occur and influence the term d in the product $K\frac{S^2}{d^2}$. One manner of deposit of the flocculated matter in the bed could be that the particles coat the surfaces of the sand grains rather evenly and thus have the effect of increasing the diameter of the sand grains. This in turn would decrease $K\frac{S^2}{d^2}$. If the floc particles were to deposit in the bed in such a manner that they did not coat the sand grains, but coalesced in the pores to form separate particles of flocculated matter, larger than the individual floc particles in the water, but not so large as the sand grains, then they would act like sand grains. The average diameter of the particles and the sand grains would lower d considerably and have the effect of increasing $K\frac{S^2}{d^2}$ during the run.

It was shown above that $K\frac{S^2}{d^2}$ was 84 times larger at the beginning of the run than at the end. This indicates that d must increase during the run to decrease $K\frac{S^2}{d^2}$. Therefore, the manner of deposit must tend more toward coating the sand grains.

If diameter alone were the only variable, to decrease $K\frac{S^2}{d^2}$ to 1/84th of its initial value, d would have to increase to a value equal to the square root of 84, or 9.16 times its original value. It would be impossible for the diameter of each individual sand grain in the bed to increase its diameter 9.16 times. This would mean an increase in volume equal to 770 times the original volume. But it is conceivable that flocculated matter could clog the pores between the sand grains so that no water could pass through, forming a dead space in the vicinity of the clogged pores.

The formation of dead spaces is illustrated below:





Open pores Clogged pores

If the pores happened to clog in such a manner that a dead space was formed approximately equal to nine times the diameter of a sand grain, the effect might be the same as increasing the diameter of a sand grain nine times, thus creating what might be called an effective diameter equal to nine times the actual diameter of the sand grains. What might conceivably happen is that dead spaces of many varying sizes might form and have an average size around nine times the diameter of the original sand grains.

The largest part of the variation of $K\frac{S^2}{d^2}$ must be caused by the factor d. Only by increasing many times its initial value can such a large decrease in K_1 be accounted for. K cannot decrease below a value of 4, nor S below a value of 6 according to the manner in which they were derived.[1] They may possibly increase, as discussed on page 20. The magnitude of the changes in K and S is not known, nor is the effect of consolidation known. In the light of present knowledge, it seems that the greatest portion of the variation in $K\frac{S^2}{d^2}$ must be brought about by a large increase in the effective diameter of the sand grains.

The rate of increase of lost head with time is practically constant throughout the length of the run for each layer of the filter.

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1. Fair, G. M. and Hatch, L. P. Journ. Am. Water Works Assn. 25: p. 1555, Nov. 1933.

The graphs of loss of head vs. time for Run 6 (Fig. 12) show how the rate of increase of lost head is practically constant for each layer throughout the whole run. In Run 8 (Fig. 15) the maximum deviations from a constant rate of increase of lost head during the course of these experiments, may be observed. One of the reasons for this is a change in the size of the floc applied to the filter. However, a tendency toward straight lines is noticed even in this run. There must be some fundamental law back of this phenomenon since it occurs so consistently throughout these experiments.

In the top layer, in which the iron removal continually decreases, the lost head builds up at a constant time rate. Even when the removal is so slight that tests cannot detect it, this rate remains constant. In the second layer, the iron removal increases for awhile, then decreases steadily. Here again, despite this variation in the rate of removal of iron, the time rate of increase of lost head ($\frac{dh}{dt}$) has a constant value throughout the length of the run. In the lower layers, the rate of removal of iron increases during the run. Each layer has a different value of $\frac{dh}{dt}$, but this value remains practically a constant for the whole run. These results seem to indicate that $\frac{dh}{dt}$ in each layer is not proportional to the rate of removal of flocculated matter from the water. However, this statement is based on the supposition that the rate of removal is the only factor influencing $\frac{dh}{dt}$. This is not true. The important factors affecting $\frac{dh}{dt}$ which have been neglected are the porosity of the filter and the rate of change of porosity with time. Towards the end of the run, when the first and second layers are approaching saturation, the effect of the decreasing porosity is far greater than the very small rate of removal. This was shown to be true

in a previous discussion (p.69). Therefore, in order to make comparisons of the effect of the rate of removal on the time rate of increase of lost head, the porosity of the filter must be the same, or nearly the same, for each condition under consideration.

Other factors not changing, for any given porosity it seems rational to expect that the time rate of increase of lost head through any portion of the filter should be proportional to the rate of deposit of flocculated matter in that portion of the filter. Furthermore, it seems reasonable to assume that the amount of matter which will deposit in the filter per unit time is proportional to the amount of matter in the water capable of being deposited. Therefore, if other factors do not change, for any given porosity it seems rational to expect that the rate of increase of lost head with time will be proportional to the concentration of flocculated matter in the water applied to the filter.

Runs 9 and 10 have an increase of iron content in the water applied to the filter during the run. The average size of floc particles remains constant during each run. The effect of the change in the iron content of the water on the rate of increase of total lost head in the filter will be considered. Before any comparisons may be made, it will be necessary to investigate the porosity of the filter to make sure that the average porosity of the whole filter has not changed appreciably between the two periods in which the comparison will be made.

A means for estimating the average porosity in the filter from the curves of iron content vs. depth in the filter (Fig. 19) must first be devised. At time A in Run 6, 85 hours, the total amount of iron removed in the top layer, as read from the mass curve, was 10.8 parts per million x hours (ppm.-hrs.). The porosity at this time was assumed as 5 percent, which meant a volume of deposited matter equal to 35.8 percent (40.8-5.0) of the total volume of the filter. Thus, 10.8 ppm - hrs. gave a volume of 35.8 percent in 0.14 feet depth. In terms of units 1 ppm - hr./foot of depth gave an increase of 0.46 percent. This means that if one part per million of iron were removed for one hour in a one-foot layer of the filter, an increase in the volume of deposited matter equal to 0.46 percent of the total volume of the filter would result. This value will be used in all the computations for porosity which follow. The change in porosity from the middle of one period to the middle of the other period under consideration will be computed. This change might be considered as the difference between the average porosities during each period. If this difference is more than five percent, the porosity difference may be too great and a comparison may not be valid.

Run 9

First period - 5 to 35 hours

From Fig. 19, the average iron removal in the entire filter from 0 to 20 hours equals 0.33 ppm.

$$\frac{0.33 \times 20}{1.96} = 3.36 \text{ ppm - hrs./ft. depth}$$

$3.36 \times 0.46 = 1.6\%$ Porosity = 40.8 - 1.6 = 39.2%

Second period 50 - 80 hrs.

Average removal from 0 to 65 hrs. equals 0.38 ppm. (Fig. 19)

$$\frac{0.38 \times 65}{1.96} \times 0.46 = 5.8\%$$

Porosity = 40.8 - 5.8 = 35.0%

The difference in porosity equals 4.2%. This is a little high, but the porosities themselves are relatively high, so the conditions in the filter as a whole, as regards porosity, will be assumed nearly alike for both periods.

Run 10

First period 5 - 35 hrs.

Average removal 0 - 20 hrs. = 0.20 ppm (Fig. 21)

$$\frac{0.20 \times 20}{1.96} \times 0.46 = 0.94\%$$

Porosity = 39.9%

Second period 60 - 90 hrs.

Average removal 0 - 75 hrs. = 0.24 ppm. (Fig. 21)

$$\frac{0.24 \times 0.75}{1.96} \times 0.46 = 4.2\%$$

Porosity = 36.6%

The difference in porosity equals 3.3%. This difference is relatively low, so the conditions in the filter as a whole, as regards porosity, will be assumed nearly alike for both periods.

Run 9

Concentration changes at 33 hours.

35 hrs. L.H. 2.67

5 hrs. L.H. 1.36

30 hrs. increase 1.31

Iron Content = 0.40 ppm.

$\frac{dh}{dt} = \frac{1.31}{30} = 0.0437'/hr.$

80 hrs. L.H. 5.20

50 hrs. L.H. 3.45

30 hrs. increase 1.75

Iron Content = 0.50 ppm.

$\frac{dh}{dt} = \frac{1.75}{30} = 0.055'/hr.$

The ratio of iron content to $\frac{dh}{dt}$ for each period equals:

$$\frac{0.40}{0.0437} = 9.15 \qquad \frac{0.50}{0.055} = 9.10$$

Run 10

Concentration changes at 37 hours.

35 hrs. L.H. 2.17

5 hrs. L.H. 1.28

30 hrs. increase 0.89

Iron Content = 0.25 ppm.

$\frac{0.89}{30} = 0.0297'/hr.$

90 hrs. L.H. 4.46

60 hrs. L. H. 3.16

30 hrs. increase 1.30

Iron Content = 0.35 ppm.

$\frac{1.30}{30} = 0.0433'/hr.$

$$\frac{0.25}{0.0297} = 8.4 \qquad \frac{0.35}{0.0433} = 8.1$$

The close agreement between the rates of increase of lost head $\left(\frac{dh}{dt}\right)$ in different parts of the run with the iron content of the applied water tends to indicate that $\frac{dh}{dt}$ is directly proportional to the iron content of the water applied to the filter. The data are too meager to accept as complete proof. In order to prove this adequately, experiments would have to be carried out, keeping other factors as constant as possible and varying the concentration for different runs, but keeping it constant during any one run.

The water applied to the filter in the latter part of Run 10 and during Run 12 has the same iron content and the same average floc size. If the iron content of the water and the average size of floc particles are a proper measure of the characteristics of the flocculated matter in the water, $\frac{dh}{dt}$ should be equal in both cases provided the porosity is nearly the same in both periods under consideration.

Run 12 30 - 60 hrs.

Aver. removal from 0 to 45 hrs. = 0.30 ppm. (Fig.23)

$$\frac{0.30 \times 45}{1.96} \times 0.46 = 3.2\%$$

Porosity = 37.6%

The porosity for the second period (60-90 hrs.) of Run 12 was found to be equal to 36.6%. These porosities are very nearly alike and will permit a comparison of results.

Run 10	90 hrs.	L.H.	4.46	$\frac{dh}{dt} = \frac{1.30}{30} = 0.0433'/hr$.
	60 hrs.	L.H.	3.16'	
	30 hrs.	increase	1.30'	
Run 12	60 hrs.	L.H.	3.70	$\frac{dh}{dt} = \frac{1.30}{30} = 0.0433'/hr.$
	30 hrs.	L.H.	2.40	
	30 hrs.	increase	1.30'	

This identity of $\frac{dh}{dt}$ for two different runs proves one of two things: either no factors which may influence $\frac{dh}{dt}$ have been neglected, or if they have been neglected, they are alike in the two runs, 10 and 12. The depth of bed, the average size of floc particles, the iron content of the water, and the temperature of the water are the same in both cases, since these have all been measured. The average porosity of the entire bed has been computed to be nearly the same in both cases. The strength of the floc particles is probably the same in both cases, although we have no measure of it, because they are composed of the same type of matter, and are about the same size. The close agreement of the effect of concentration on $\frac{dh}{dt}$ obtained in previous computations also shows that no factors influencing $\frac{dh}{dt}$ have been neglected, or, if neglected, they are the same for both runs.

The mass curve of total iron removed in the top layer of the filter in Run 6 is given in Fig. 28. Its units are in parts per million x hours. Knowing the quantity of water flowing per unit time and the total amount of iron removed in parts per million x hours, it is possible to compute the total weight of iron deposited during the whole of Run 6 in the top layer of the filter as follows: The total iron removed was 10.8 ppm - hrs. Over a period of 80 hours the average iron removal in the layer was $\frac{10.8}{80}$, or 0.135 ppm. The rate of flow was 2 gallons per square foot per minute. This is equal to $2 \times 3.785 \times 10^3 \times 60$, or 4.55×10^5 cc./sq.ft./hr. The weight of water flowing per hour was 4.55×10^5 grams/sq.ft/hr. In 80 hours, $4.55 \times 10^5 \times 80$, or 3.64×10^7 grams of water have passed by each square foot of filter area. Out of each million grams of water, 0.135 grams of iron have been removed by the top layer. Therefore, the total

weight of iron removed per square foot of filter area equals $3.64 \times 10^7 \times 0.135 \times 10^6$, or 4.91 grams of iron.

As explained on page 32, the iron content of the water was found to be approximately equal to 8.5 percent of the dry weight of suspended solids in the water. Using this figure, the dry weight of suspended solids deposited in the top layer during Run 6 may be computed. 4.91 grams of iron were removed. This is equivalent to $\frac{4.91}{.085}$, or 57.8 grams of suspended matter removed per square foot of the top layer.

Knowing the average porosity of the layer, the volume of deposited matter may be computed. Porosity has been assumed as 5 percent. The depth of the top layer is 0.14 feet. The volume of deposit per square foot of filter area is equal to $(0.408 - 0.05) \times 0.14 = 0.050$ cubic feet. This is equal to 0.050×28320, or 1416 c.c. of flocculated matter. Since the flocculated matter is mostly composed of water, its specific gravity may be assumed as 1.0. Therefore, 1416 grams of flocculated matter have been deposited per square foot of the top layer. The suspended solids content of this matter will be equal to $\frac{57.8}{1416} \times 100$, or approximately 4 percent. Therefore, the water content of the flocculated matter in the filter is equal to 96 percent.

This result is based on the assumption of a porosity in the top layer equal to 5 percent near the end of the run. The value of 96 percent, therefore, may not be accurate. The computations and results are presented here to illustrate the method with which to compare the results of iron tests with those obtained by weighing the deposit in

the units of the small filter. Extended research will have to be carried out to determine the relation between iron content and suspended solids in the water over a wide range of values of iron content and suspended solids, in order to provide an accurate means of checking the results derived from the iron tests with those obtained by weighing.

10. CONCLUSIONS

From the results of these experiments, the following conclusions were drawn:

1. Contrary to general opinion, the entire filter bed shares the burden of removal of flocculated matter from the water. The upper portion of the bed performs most of the removal at the beginning of the run, but as the upper pores become filled with floc, the burden is taken by the lower portions of the filter. It is erroneous to assume that depth has no influence on the performance of the filter.

2. The top layer of the filter approaches a saturation point. The curve of removal of solid matter from the water vs. time becomes asymptotic to the zero-removal axis, and the removal becomes a very small quantity. But this very small amount of removal has a considerable effect on the porosity and on the rate of increase of lost head with time.

3. When the top layer of the filter is near its saturation point, the factors K, S, and d in the equation of filtration do not vary. The porosity is the all-important factor in this region.

4. The product $K\frac{S^2}{d^2}$ varies considerably during the run, decreasing many times more than can be accounted for by an increase in the diameter of each individual sand grain.

5. The manner of deposit of flocculated matter in the filter tends more toward coating the sand grain in order to have the effect of increasing the effective diameter of the grains.

6. If the effect of porosity is negligible, the time rate of increase of lost head increases with an increase in the concentration of flocculated matter in the water, and appears to be directly proportional to it, other things remaining the same.

APPENDIX

TABLE 5

DATA FOR DETERMINATION OF THE SHAPE FACTOR

Piez. No.	Dist. apart of Piez.	Dist. from Bottom	Manometer Readings			
1		0.0	6.915	6.882	6.833	6.810
2	7.80	7.80	6.512	6.458	6.374	6.329
3	1.48	9.28	6.440	6.387	6.292	6.245
4	4.50	13.78	6.228	6.161	6.043	5.983
5	1.49	15.27	6.157	6.086	5.959	5.897
6	1.50	16.77	6.087	6.015	5.880	5.812
7	1.50	18.27	6.016	5.937	5.800	5.727
8	1.54	19.81	5.948	5.867	5.718	5.641
9	1.50	21.31	5.888	5.795	5.640	5.558
10	1.50	22.81	5.812	5.720	5.580	5.476
11	1.49	24.30	5.744	5.654	5.484	5.393
12	1.48	25.78	5.679	5.584	5.407	5.314
Temperature °F			36.5	36.9	37.9	38.5
Rate cc/min.			895	800	620	537
Total Depth			42.83"	40.27"	35.90"	33.87"

COMPUTATIONS FOR THE SHAPE FACTOR

The following equations are utilized:

$$\frac{h}{l_e} = \frac{R-r}{r}\,(1-f_e) \qquad (2)$$

$$\frac{f_e^3}{1-f_e} = \frac{K\mu}{R-r}\,V\,\frac{S^2}{d^2}$$

These factors are explained on Page 19. The wash at the rate of 895 cc. per minute will be used to compute the shape factor.

Rate = 895 cc./min. $\qquad V = \frac{895}{60 \times 9.09} = 1.641$ cm/sec.

Area of filter = 9.09 sq. cm.

The sand between piezometers 4 and 12 will be considered. From Table 5, the manometer readings are obtained and the distance from the bottom of the filter to each piezometer may be read.

h = 6.229 - 5.679 = 0.550'

le = 25.78 - 13.78 = 12.00" 1.00'

Using Equation 2: $\frac{0.550}{1.00} = 1.65\,(1 - f_e)$

$1 - f_e = 0.333$, $f_e = 0.667$. This is the value of the porosity of the expanded bed at the rate of wash of 895 cc. per minute.

The geometric mean diameter of the sand grains between these piezometers must be determined. This will be equal to the square root of the product of the diameters at each of these depths. The diameter may be read from the sieve analysis curve if the weight of the sand above each of these piezometers is known. Since the lost head in washing

is equal to the weight in water of the grains suspended, this gives the necessary information.

The lost head between the bottom of the filter and piezometer 12 equals 6.915 - 5.679 = 1.236'. This lost head is equal to the weight in water of the sand grains suspended in this section. Thus, the weight of the sand is: $1.236 \times 30.48 \times 9.09 \times \frac{2.65}{1.65}$, or 550 grams. The total weight of sand grains in the filter equals 844.5 grams. $\frac{550}{844.5}$ equals 65.1 percent of the grains between the bottom of the filter and piezometer 4. This means that 34.9 percent of the total weight of grains are above this piezometer. From the sieve analysis curve (Fig. 4), the diameter corresponding to this percent weight is found to be equal to 0.515 mm.

The lost head between the bottom of the filter and piezometer 12 equals 6.915 - 6.229, or 0.686 feet. In a similar manner to that described above, the diameter at piezometer 12 is found to be equal to 0.562 mm. The geometric mean diameter equals $\sqrt{0.515 \times 0.562} = 0.54$mm.

$$T = 36.5^{\circ} F \qquad \mu = 1.68 \times 10^{-5} \text{ (Fig. 3)}$$

Using Equation 3:

$$\frac{(0.667)^3}{0.333} = 4 \times \frac{1.68 \times 10^{-5}}{1.65} \times 1.641 \times \frac{S^2}{d^2}$$

$$\left(\frac{S}{d}\right)^2 = 13360 \qquad d = 0.054 \text{ cm.}$$

$$S^2 = 38.9 \qquad S = 6.23$$

Performing the same calculations for a rate of wash of 800 cc/min., using the section between piezometers 4 and 12, the shape factor is found to be equal to 6.17. In like manner, using a rate of wash of 537 cc/min., and the section between piezometers 3 and 11, a shape factor equal to 6.20 is obtained. Averaging these three values, the shape factor is found to be equal to 6.20.

TABLE 6

CALIBRATION OF SIEVES BY COUNT AND WEIGH METHOD

Sieve Opening mm.	100 grains Wt. in gms.	Wt. per grain-gms.	$\sqrt[3]{\frac{6}{\pi}\frac{w}{\gamma g}}$ mm
0.833	0.0890	0.000890	0.863
0.701	0.0654	0.000654	0.778
0.589	0.0410	0.000410	0.666
0.495	0.0224	0.000224	0.545
0.417	0.0134	0.000134	0.459
0.351	0.0104	0.000104	0.422

TABLE 7

LOST HEAD IN FEET IN VARIOUS LAYERS OF THE FILTER

RUN 6

Time	Layer				
Hours	1	2	3	4	5
4	1.34	1.07	0.80	0.47	0.28
7	1.49	1.21	0.93	0.60	0.38
10.5	1.65	1.36	1.08	0.73	0.48
23	2.27	1.98	1.69	1.29	0.87
26	2.41	2.12	1.82	1.38	0.94
31.5	2.72	2.41	2.10	1.65	1.12
36	2.90	2.60	2.26	1.79	1.19
47.5	3.52	3.21	2.87	2.32	1.50
54	3.90	3.57	3.22	2.61	1.68
58	4.12	3.79	3.41	2.78	1.76
71	4.89	4.57	4.15	3.39	2.13
76.5	5.11	4.77	4.34	3.52	2.19
82	5.52	5.14	4.70	3.80	2.36
95.5	6.36	6.00	5.54	4.45	2.72
101.5	6.60	6.34	5.81	4.65	2.80

TABLE 8

IRON CONTENT OF WATER IN PARTS PER MILLION
AT VARIOUS DEPTHS IN THE FILTER

RUN 6

Time	Depth in Filter					
Hours	Infl.	0.14'	0.39'	0.89'	1.39'	Eff.
2	0.60	0.22	0.11	0.08	0.07	0.07
12	0.60	0.32	0.20	0.09	0.07	0.06
24	0.60	0.42	0.28	0.14	0.10	0.07
36	0.60	0.48	0.35	0.18	0.12	0.09
48	0.60	0.52	0.42	0.23	0.16	0.12
58	0.60	0.56	0.48	0.27	0.19	0.15
72	0.60	0.59	0.53	0.31	0.23	0.19
83	0.60	0.60	0.56	0.33	0.24	0.20
96	0.60	0.60	0.58	0.34	0.25	0.20

TABLE 9

LOST HEAD IN FEET IN VARIOUS LAYERS OF THE FILTER

RUN 8

Time	Layer				
Hours	1	2	3	4	5
3	1.32	1.01	0.76	0.47	0.27
8	1.48	1.18	0.93	0.58	0.35
22	2.09	1.78	1.50	1.09	0.73
26	2.27	1.95	1.68	1.27	0.84
30.5	2.46	2.15	1.86	1.41	0.94
35	2.67	2.36	2.06	1.60	1.04
46	3.20	2.88	2.57	2.05	1.32
50.5	3.51	3.17	2.86	2.28	1.48
55	3.64	3.41	3.09	2.48	1.58
70	4.44	4.10	3.77	3.04	1.92
75	4.71	4.34	3.98	3.23	2.06
81	5.03	4.66	4.29	3.49	2.22
94	5.88	5.49	5.08	4.19	2.63
100	6.32	5.93	5.51	4.55	2.87
108	6.86	6.45	6.01	4.95	3.11
118	7.60	7.17	6.69	5.50	3.47

TABLE 10

IRON CONTENT OF WATER IN PARTS PER MILLION
AT VARIOUS DEPTHS IN THE FILTER

RUN 8

Time	Depth in Filter					
Hours	Infl.	0.14'	0.39'	0.89'	1.39'	Eff.
9	0.50	0.20	0.12	0.07	0.05	0.05
23	0.50	0.32	0.20	0.08	0.06	0.05
36	0.50	0.40	0.27	0.10	0.08	0.07
47	0.50	0.45	0.32	0.15	0.12	0.10
55	0.50	0.49	0.37	0.18	0.15	0.13
70	0.50	0.50	0.40	0.22	0.18	0.15
82	0.50	0.50	0.42	0.25	0.19	0.16
95	0.50	0.50	0.44	0.28	0.20	0.17
109	0.50	0.50	0.46	0.30	0.20	0.17
119	0.50	0.50	0.48	0.34	0.22	0.18

TABLE 11

LOST HEAD IN FEET IN VARIOUS LAYERS OF THE FILTER

RUN 9

Time	Layer				
Hours	1	2	3	4	5
2	1.23	0.96	0.70	0.36	0.17
6.5	1.43	1.12	0.87	0.52	0.30
20	1.99	1.68	1.43	1.06	0.71
28	2.35	2.06	1.80	1.36	0.95
32	2.51	2.22	1.93	1.49	1.02
45	3.14	2.85	2.55	2.03	1.41
49	3.40	3.10	2.80	2.26	1.55
68	4.53	4.21	3.91	3.21	2.22
73	4.78	4.46	4.12	3.48	2.37
78	5.06	4.74	4.39	3.68	2.53
94	6.12	5.80	5.40	4.56	3.17
99.5	6.56	6.22	5.83	4.92	3.41

TABLE 12

IRON CONTENT OF WATER IN PARTS PER MILLION
AT VARIOUS DEPTHS IN THE FILTER

RUN 9

Time	Depth in Filter					
Hours	Infl.	0.14'	0.39'	0.89'	1.39'	Eff.
7	0.40	0.18	0.08	0.07	0.07	0.07
21	0.40	0.30	0.15	0.08	0.07	0.07
33	0.45	0.36	0.19	0.09	0.07	0.06
45	0.50	0.42	0.23	0.11	0.07	0.06
55	0.50	0.45	0.26	0.12	0.07	0.06
68	0.50	0.47	0.28	0.14	0.08	0.07
79	0.50	0.49	0.32	0.14	0.10	0.07
94	0.50	0.50	0.36	0.15	0.12	0.10

TABLE 13

LOST HEAD IN FEET IN VARIOUS LAYERS OF THE FILTER

RUN 10

Time Hours	Layer 1	2	3	4	5
2	1.20	0.90	0.63	0.32	0.17
15	1.57	1.25	0.97	0.64	0.42
19	1.68	1.37	1.08	0.75	0.52
25.5	1.87	1.55	1.26	0.92	0.65
36.5	2.22	1.89	1.60	1.24	0.92
43	2.45	2.12	1.84	1.46	1.09
46	2.60	2.27	1.98	1.58	1.17
49	2.71	2.38	2.09	1.68	1.26
62.5	3.24	2.93	2.62	2.18	1.62
68	3.48	3.15	2.85	2.38	1.75
74.5	3.75	3.42	3.12	2.62	1.94
84	4.19	3.87	3.54	3.01	2.23
97	4.76	4.44	4.10	3.54	2.54

TABLE 14

IRON CONTENT OF WATER IN PARTS PER MILLION
AT VARIOUS DEPTHS IN THE FILTER

RUN 10

Time Hours	Depth in Filter Infl.	0.14'	0.39'	0.89'	1.39'	Eff.
2	0.25	0.09	0.06	0.05	0.05	0.05
15	0.25	0.13	0.07	0.06	0.05	0.05
26	0.25	0.17	0.08	0.06	0.05	0.05
37	0.30	0.24	0.10	0.07	0.06	0.06
49	0.35	0.28	0.13	0.08	0.06	0.06
63	0.35	0.30	0.15	0.08	0.06	0.06
75	0.35	0.32	0.19	0.08	0.06	0.06
85	0.35	0.33	0.21	0.10	0.07	0.07

TABLE 15

LOST HEAD IN FEET IN VARIOUS LAYERS OF THE FILTER

RUN 12

Time	Layer				
Hours	1	2	3	4	5
3	1.27	0.94	0.67	0.34	0.15
9	1.55	1.23	0.94	0.60	0.37
21	1.99	1.66	1.38	1.02	0.71
25	2.16	1.83	1.55	1.17	0.82
31	2.40	2.06	1.77	1.40	1.00
34	2.54	2.20	1.93	1.54	1.11
45	3.03	2.68	2.39	1.95	1.44
50	3.23	2.89	2.59	2.15	1.59
57	3.52	3.19	2.88	2.42	1.77
68	4.06	3.72	3.40	2.89	2.11
74.5	4.36	4.01	3.69	3.18	2.28
79.5	4.60	4.26	3.92	3.38	2.43
91	5.25	4.90	4.55	3.93	2.76
96	5.55	5.20	4.83	4.21	2.98
99.5	5.77	5.41	5.04	4.38	3.09
104	6.02	5.66	5.33	4.63	3.24
117.5	6.84	6.47	6.08	5.30	3.63
123	7.24	6.85	6.46	5.63	3.82
129	7.60	7.20	6.80	5.92	3.99
137.5	8.17	7.77	7.35	6.42	4.30

TABLE 16

IRON CONTENT OF WATER IN PARTS PER MILLION
AT VARIOUS DEPTHS IN THE FILTER

RUN 12

Time	Depth in Filter					
Hours	Infl.	0.14'	0.39'	0.89'	1.39'	Eff.
4	0.35	0.11	0.07	0.06	0.05	0.05
22	0.35	0.19	0.10	0.06	0.04	0.04
32	0.35	0.23	0.12	0.07	0.05	0.05
46	0.35	0.26	0.15	0.08	0.05	0.05
58	0.35	0.28	0.18	0.09	0.06	0.05
69	0.35	0.30	0.21	0.10	0.07	0.05
80	0.35	0.32	0.23	0.11	0.07	0.05
92	0.35	0.34	0.26	0.12	0.07	0.05
105	0.35	0.34	0.28	0.13	0.07	0.05
118	0.35	0.35	0.30	0.14	0.08	0.06
138	0.35	0.35	0.33	0.16	0.10	0.08

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